The Dow-Hart collaboration

Luxury!

K. P. Hart

Faculty EEMCS
TU Delft

Ithaca, 6 December 2014: 14:30 – 15:30

Outline

Delft University of Technology

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist

Delft University of Technology

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist
2. (archaic) to have worth, value, validity, availability, or suitableness

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist
2. (archaic) to have worth, value, validity, availability, or suitableness
3. (chiefly Scot) to be able or capable

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist
2. (archaic) to have worth, value, validity, availability, or suitableness
3. (chiefly Scot) to be able or capable
4. to endow or give as an endowment

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist
2. (archaic) to have worth, value, validity, availability, or suitableness
3. (chiefly Scot) to be able or capable
4. to endow or give as an endowment
5. (chiefly Scot) to fade away

Delft University of Technology

Dow

1. an Arab lateenrigged boat of the Indian ocean usu. having a long overhang forward, a high poop, and an open waist
2. (archaic) to have worth, value, validity, availability, or suitableness
3. (chiefly Scot) to be able or capable
4. to endow or give as an endowment
5. (chiefly Scot) to fade away
6. a crooked knife

Delft University of Technology

Outline

Delft University of Technology

What is a continuum?

Chambers: that which is continuous; that which must be regarded as continuous and the same and which can be described only relatively.

What is a continuum?

Chambers: that which is continuous; that which must be regarded as continuous and the same and which can be described only relatively.

Oxford Canadian Dictionary: anything as having a continuous structure without perceptibly distinct parts

Delft University of Technology

What is a continuum?

Chambers: that which is continuous; that which must be regarded as continuous and the same and which can be described only relatively.

Oxford Canadian Dictionary: anything as having a continuous structure without perceptibly distinct parts

For us: a compact and connected Hausdorff space

Our favourites

$\mathbb{H}^*$: the Čech-Stone remainder of the half line $\mathbb{H}$, where $\mathbb{H} = [0, \infty)$

Our favourites

$\mathbb{H}^*$: the Čech-Stone remainder of the half line $\mathbb{H}$, where $\mathbb{H} = [0, \infty)$

$\mathbb{I}_u$: preimage of u under $\beta\pi$, where $\pi : \omega \times \mathbb{I} \to \omega$ is the natural map (and $\mathbb{I} = [0, 1]$)

Our favourites

$\mathbb{H}^*$: the Čech-Stone remainder of the half line $\mathbb{H}$, where $\mathbb{H} = [0, \infty)$

$\mathbb{I}_u$: preimage of u under $\beta\pi$, where $\pi : \omega \times \mathbb{I} \to \omega$ is the natural map (and $\mathbb{I} = [0, 1]$)

(We write $\mathbb{M} = \omega \times \mathbb{I}$)

Outline

Delft University of Technology

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$

Delft University of Technology

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$, as they have elementarily equivalent bases for the closed sets

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$, as they have elementarily equivalent bases for the closed sets, until you take a closer look.

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$, as they have elementarily equivalent bases for the closed sets, until you take a closer look.

There are cut points: if $\langle x_n \rangle_n$ is a sequence in $\mathbb{I}$ then x_u, the unique point in $\mathbb{I}_u$ that is in the closure of $\{\langle n, x_n \rangle : n \in \omega\}$, is a cut point.

Delft University of Technology

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$, as they have elementarily equivalent bases for the closed sets, until you take a closer look.

There are cut points: if $\langle x_n \rangle_n$ is a sequence in $\mathbb{I}$ then x_u, the unique point in $\mathbb{I}_u$ that is in the closure of $\{\langle n, x_n \rangle : n \in \omega\}$, is a cut point.

There are non-cut points too, e.g., the supremum of a countable increasing sequence of cut points.

Delft University of Technology

Cut points

If $u \in \omega^*$ then $\mathbb{I}_u$ kind of looks like $\mathbb{I}$, as they have elementarily equivalent bases for the closed sets, until you take a closer look.

There are cut points: if $\langle x_n \rangle_n$ is a sequence in $\mathbb{I}$ then x_u, the unique point in $\mathbb{I}_u$ that is in the closure of $\{\langle n, x_n \rangle : n \in \omega\}$, is a cut point.

There are non-cut points too, e.g., the supremum of a countable increasing sequence of cut points.

(The speaker had better draw a few instructive pictures at this point.)

Any other cut points?

Good question.

Any other cut points?

Good question.

Continuum Hypothesis implies yes. They can live on very thin and very thick sets.

Delft University of Technology

Any other cut points?

Good question.

Continuum Hypothesis implies yes. They can live on very thin and very thick sets.

Take a copy, F_n, of the ordinal $\omega^n + 1$ in $\mathbb{I}$

Delft University of Technology

Any other cut points?

Good question.

Continuum Hypothesis implies yes. They can live on very thin and very thick sets.

Take a copy, F_n, of the ordinal $\omega^n + 1$ in $\mathbb{I}$; there is a non-trivial cut point in the closure of $\bigcup_n \{n\} \times F_n$.

Any other cut points?

Good question.

Continuum Hypothesis implies yes. They can live on very thin and very thick sets.

Take a copy, F_n, of the ordinal $\omega^n + 1$ in $\mathbb{I}$; there is a non-trivial cut point in the closure of $\bigcup_n \{n\} \times F_n$.

There is also one that is not in the closure of any nowhere dense subset of $\mathbb{M}$

Any other cut points?

Good question.

Continuum Hypothesis implies yes. They can live on very thin and very thick sets.

Take a copy, F_n, of the ordinal $\omega^n + 1$ in $\mathbb{I}$; there is a non-trivial cut point in the closure of $\bigcup_n \{n\} \times F_n$.

There is also one that is not in the closure of any nowhere dense subset of $\mathbb{M}$ (did somebody say "remote point"?).

Any other cut points?

In the Laver model there are no non-trivial cut points.

Any other cut points?

In the Laver model there are no non-trivial cut points.

Why?

Any other cut points?

In the Laver model there are no non-trivial cut points.

Why?

Because: x is a cut point iff for every $f : \omega \to \mathbb{N}$ there is $g : \omega \to \mathbb{N}$ such that $g(n) < f(n)$ for all n and x is in the closure of $\bigcup_n \{n\} \times \left[\frac{g(n)}{f(n)}, \frac{g(n)+1}{f(n)}\right]$.

Any other cut points?

In the Laver model there are no non-trivial cut points.

Why?

Because: x is a cut point iff for every $f : \omega \to \mathbb{N}$ there is $g : \omega \to \mathbb{N}$ such that $g(n) < f(n)$ for all n and x is in the closure of $\bigcup_n \{n\} \times \left[\frac{g(n)}{f(n)}, \frac{g(n)+1}{f(n)}\right]$.

The Laver reals dash the hopes of any aspiring non-trivial cut points.

A question

Let L be Lokucievskiĭ's compact space. It satisfies $\dim L = 1$ and $\operatorname{ind} L = \operatorname{Ind} L = 2$

A question

Let L be Lokucievskiĭ's compact space. It satisfies $\dim L = 1$ and $\operatorname{ind} L = \operatorname{Ind} L = 2$

Under the Continuum Hypothesis we have
$\dim L_u = \operatorname{ind} L_u = \operatorname{Ind} L_u = 1$.

A question

Let L be Lokucievskiĭ's compact space. It satisfies $\dim L = 1$ and $\operatorname{ind} L = \operatorname{Ind} L = 2$

Under the Continuum Hypothesis we have $\dim L_u = \operatorname{ind} L_u = \operatorname{Ind} L_u = 1$.

Question

What is $\operatorname{Ind} L_u$ in the Laver model?

Esurient

pres. part. of *esurire* to be hungry, desiderative of *edere* to eat

Esurient

pres. part. of *esurire* to be hungry, desiderative of *edere* to eat

desiderative: of a verb or verb form: derived from or belonging to the inflectional paradigm of a verb and expressing a desire to perform the action denoted by that verb — the Latin verb *esurire* "to be hungry" is from *edere* "to eat"

Esurient

pres. part. of *esurire* to be hungry, desiderative of *edere* to eat

desiderative: of a verb or verb form: derived from or belonging to the inflectional paradigm of a verb and expressing a desire to perform the action denoted by that verb — the Latin verb *esurire* "to be hungry" is from *edere* "to eat"

Question

What about higher-order desideratives?

Delft University of Technology

Outline

Continuous images

A classic: every metrizable compact space is a continuous image of the Cantor set.

Continuous images

A classic: every metrizable compact space is a continuous image of the Cantor set.

Another classic (in certain circles): every compact Hausdorff space of weight at most $\aleph_1$ is a continuous image of ω^*.

Delft University of Technology

Continuous images

A classic: every metrizable compact space is a continuous image of the Cantor set.

Another classic (in certain circles): every compact Hausdorff space of weight at most $\aleph_1$ is a continuous image of ω^*.

Another classic (in certain other circles): there is no metric continuum that maps onto every metric continuum.

Continuous images

A classic: every metrizable compact space is a continuous image of the Cantor set.

Another classic (in certain circles): every compact Hausdorff space of weight at most $\aleph_1$ is a continuous image of ω^*.

Another classic (in certain other circles): there is no metric continuum that maps onto every metric continuum.

On its way to becoming a classic: every continuum of weight at most $\aleph_1$ is a continuous image of $\mathbb{H}^*$.

Continuous images

The proof of the classic-in-the-making uses elementarity to overcome difficulties that do not seem to occur in the classic case.

Continuous images

The proof of the classic-in-the-making uses elementarity to overcome difficulties that do not seem to occur in the classic case.

Elementarity is present in the case of images of ω^*: all atomless Boolean algebras are elementarily equivalent.

Continuous images and CH

We have:

Continuous images and CH

We have:

Compact metrizable iff image of the Cantor set

Continuous images and CH

We have:

Compact metrizable iff image of the Cantor set

Compact weight at most $\mathfrak{c}$ iff image of ω^*

Continuous images and CH

We have:

Compact metrizable iff image of the Cantor set

Compact weight at most $\mathfrak{c}$ iff image of ω^*

Metrizable continuum iff ...

Delft University of Technology

Continuous images and CH

We have:

Compact metrizable iff image of the Cantor set

Compact weight at most $\mathfrak{c}$ iff image of ω^*

Metrizable continuum iff ...

Continuum of weight at most $\mathfrak{c}$ iff image of $\mathbb{H}^*$.

Delft University of Technology

Continuous images and CH

We have:
Compact metrizable iff image of the Cantor set
Compact weight at most $\mathfrak{c}$ iff image of ω^*
Metrizable continuum iff . . .
Continuum of weight at most $\mathfrak{c}$ iff image of $\mathbb{H}^*$.

Note that in the continuum case the uncountable seems to behave better than the countable.

Continuous images and CH

We have:
Compact metrizable iff image of the Cantor set
Compact weight at most $\mathfrak{c}$ iff image of ω^*
Metrizable continuum iff ...
Continuum of weight at most $\mathfrak{c}$ iff image of $\mathbb{H}^*$.

Note that in the continuum case the uncountable seems to behave better than the countable.

Question

Why?

More results

Consistently:

More results

Consistently:

Not every separable continuum is a continuous image of $\mathbb{H}^*$ (it *is* a continuous image of ω^*).

More results

Consistently:

Not every separable continuum is a continuous image of $\mathbb{H}^*$ (it *is* a continuous image of ω^*).

There is a first-countable continuum that is not a continuous image of $\mathbb{H}^*$.

Delft University of Technology

More results

Consistently:

Not every separable continuum is a continuous image of $\mathbb{H}^*$ (it *is* a continuous image of ω^*).

There is a first-countable continuum that is not a continuous image of $\mathbb{H}^*$.
But

TU Delft
Delft University of Technology

More results

Consistently:

Not every separable continuum is a continuous image of $\mathbb{H}^*$ (it *is* a continuous image of ω^*).

There is a first-countable continuum that is not a continuous image of $\mathbb{H}^*$.
But:

Question

Is every perfectly normal continuum a continuous image of $\mathbb{H}^*$?

Delft University of Technology

Mosey

A disturbing discovery:

Mosey

A disturbing discovery:
Originally (1829): to go away quickly or promptly; to make haste (now rare). Later (1960) usually: to walk in a leisurely or aimless manner; to amble, wander.

Mosey

A disturbing discovery:
Originally (1829): to go away quickly or promptly; to make haste (now rare). Later (1960) usually: to walk in a leisurely or aimless manner; to amble, wander.

Fortunately there is **soodle** (1821): To walk in a slow or leisurely manner; to stroll, saunter.

Delft University of Technology

Outline

1. First a definition
2. Continua
    - Cut points
    - Continuous images
    - Subcontinua of $\mathbb{H}^*$
3. More general spaces
    - Continuous images
    - Invisible spaces
    - Other stuff
4. The last word

Delft University of Technology

Easy examples

Every $\mathbb{I}_u$ is a subcontinuum of $\mathbb{H}^*$.

Delft University of Technology

Easy examples

Every $\mathbb{I}_u$ is a subcontinuum of $\mathbb{H}^*$.
Hence so are their intervals, of which there are certainly six types, depending on the end sets: point, blob (countable cofinality), blob (uncountable cofinality).

Easy examples

Every $\mathbb{I}_u$ is a subcontinuum of $\mathbb{H}^*$.
Hence so are their intervals, of which there are certainly six types, depending on the end sets: point, blob (countable cofinality), blob (uncountable cofinality).
Blobs are indecomposable, so a point and a blob give us two more.

Easy examples

Every $\mathbb{I}_u$ is a subcontinuum of $\mathbb{H}^*$.
Hence so are their intervals, of which there are certainly six types, depending on the end sets: point, blob (countable cofinality), blob (uncountable cofinality).
Blobs are indecomposable, so a point and a blob give us two more.

(Another instructive picture, perhaps?)

Delft University of Technology

Slightly more complicated examples

Number nine

Slightly more complicated examples

Number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Delft University of Technology

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Delft University of Technology

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine

Delft University of Technology

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, . . .

Slightly more complicated examples

Number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, number nine, . . .

the closure of the union of an increasing ω-sequence of indecomposable subcontinua

Slightly more complicated examples

Number ten: constructed from a minimal shift-invariant subset of ω^*.

Delft University of Technology

Slightly more complicated examples

Number ten: constructed from a minimal shift-invariant subset of ω^*.

(Instructive picture needed.)

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

- under CH: four more subcontinua of $\mathbb{H}^*$

Delft University of Technology

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

- under CH: four more subcontinua of $\mathbb{H}^*$
- under $\neg$CH: six more subcontinua of $\mathbb{H}^*$

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

- under CH: four more subcontinua of $\mathbb{H}^*$
- under $\neg$CH: six more subcontinua of $\mathbb{H}^*$

Surely 14 can not be the definitive answer?

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

- under CH: four more subcontinua of $\mathbb{H}^*$
- under $\neg$CH: six more subcontinua of $\mathbb{H}^*$

Surely 14 can not be the definitive answer?

The answer should be 42

A few more

Mucking about with intervals in $\mathbb{I}_u$s and ultrapowers will give us

- under CH: four more subcontinua of $\mathbb{H}^*$
- under $\neg$CH: six more subcontinua of $\mathbb{H}^*$

Surely 14 can not be the definitive answer?

The answer should be 42 or ...

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

If CH fails there are sufficiently many distinct $\mathbb{I}_u$s

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

If CH fails there are sufficiently many distinct $\mathbb{I}_u$s

If CH holds all $\mathbb{I}_u$s are homeomorphic but we can find two families of $2^{\aleph_1}$ continua

Delft University of Technology

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

If CH fails there are sufficiently many distinct $\mathbb{I}_u$s

If CH holds all $\mathbb{I}_u$s are homeomorphic but we can find two families of $2^{\aleph_1}$ continua, decomposable and indecomposable

Delft University of Technology

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

If CH fails there are sufficiently many distinct $\mathbb{I}_u$s

If CH holds all $\mathbb{I}_u$s are homeomorphic but we can find two families of $2^{\aleph_1}$ continua, decomposable and indecomposable (the indecomposable ones look like number nine

$2^{\mathfrak{c}}$, the expected answer

There are $2^{\mathfrak{c}}$ mutually non-homeomorphic subcontinua of $\mathbb{H}^*$.

If CH fails there are sufficiently many distinct $\mathbb{I}_u$s

If CH holds all $\mathbb{I}_u$s are homeomorphic but we can find two families of $2^{\aleph_1}$ continua, decomposable and indecomposable (the indecomposable ones look like number nine, . . .)

Congeries

Congeries: A collection of things merely massed or heaped together; a mass, heap.

Congeries

Congeries: A collection of things merely massed or heaped together; a mass, heap.

As in: a MAD family is an infinite maximal element in the *congeries* of almost disjoint families ordered by inclusion.

Delft University of Technology

Congeries

Congeries: A collection of things merely massed or heaped together; a mass, heap.

As in: a MAD family is an infinite maximal element in the *congeries* of almost disjoint families ordered by inclusion.

(Six math reviews have ‘congeries’ in them, two of the papers are by Alan.)

Outline

1. First a definition
2. Continua
   - Cut points
   - Continuous images
   - Subcontinua of $\mathbb{H}^*$
3. More general spaces
   - Continuous images
   - Invisible spaces
   - Other stuff
4. The last word

Delft University of Technology

Outline

Delft University of Technology

Images of ω^*

One of the aforementioned classics becomes, under CH

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

Delft University of Technology

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as: $\mathbb{H}^*$

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as: $\mathbb{H}^*$, Stone space of the measure algebra

Delft University of Technology

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as: $\mathbb{H}^*$, Stone space of the measure algebra, Stone space of the category algebra

Delft University of Technology

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as: $\mathbb{H}^*$, Stone space of the measure algebra, Stone space of the category algebra, long segment of length $\mathfrak{c}$, ...

Delft University of Technology

Images of ω^*

One of the aforementioned classics becomes, under CH: the continuous images of ω^* are exactly the compact Hausdorff spaces of weight at most $\mathfrak{c}$.

This includes all kinds of quite nice and concrete spaces, such as: $\mathbb{H}^*$, Stone space of the measure algebra, Stone space of the category algebra, long segment of length $\mathfrak{c}$, ...

Is CH really necessary?

Delft University of Technology

Yes!

Well, . . .

Yes!

Well, . . . , the Stone space of the category algebra is separable and hence a continuous image of ω^*.

Yes!

Well, . . . , the Stone space of the category algebra is separable and hence a continuous image of ω^*.
In the $\aleph_2$-Cohen model the long segment of length ω_2 is not a continuous image of ω^*

Delft University of Technology

Yes!

Well, ..., the Stone space of the category algebra is separable and hence a continuous image of ω^*.
In the $\aleph_2$-Cohen model the long segment of length ω_2 is not a continuous image of ω^*

The Open Colouring Axiom takes care of the other two

Delft University of Technology

Yes!

Well, . . . , the Stone space of the category algebra is separable and hence a continuous image of ω^*.
In the $\aleph_2$-Cohen model the long segment of length ω_2 is not a continuous image of ω^*

The Open Colouring Axiom takes care of the other two: it implies that neither the Stone space of the measure algebra nor $\mathbb{H}^*$

Yes!

Well, . . . , the Stone space of the category algebra is separable and hence a continuous image of ω^*.
In the $\aleph_2$-Cohen model the long segment of length ω_2 is not a continuous image of ω^*

The Open Colouring Axiom takes care of the other two: it implies that neither the Stone space of the measure algebra nor $\mathbb{H}^*$ (and a host of other remainders)

Delft University of Technology

Yes!

Well, . . . , the Stone space of the category algebra is separable and hence a continuous image of ω^*.
In the $\aleph_2$-Cohen model the long segment of length ω_2 is not a continuous image of ω^*

The Open Colouring Axiom takes care of the other two: it implies that neither the Stone space of the measure algebra nor $\mathbb{H}^*$ (and a host of other remainders) is a continuous image of ω^*.

Delft University of Technology

Agley

The best laid

Agley

The best laid schemes o' Mice an' Men, Gang aft agley

Agley

The best laid schemes o' Mice an' Men, Gang aft agley

Awry, wrong; askew; obliquely, asquint.

Agley

The best laid schemes o' Mice an' Men, Gang aft agley

Awry, wrong; askew; obliquely, asquint. Freq. to go (also gang, etc.) agley.

Delft University of Technology

Agley

The best laid schemes o' Mice an' Men, Gang aft agley

Awry, wrong; askew; obliquely, asquint. Freq. to go (also gang, etc.) agley.

A useful word for mathematicians.

Delft University of Technology

Outline

Delft University of Technology

Small diagonals

A space, X, has a *small diagonal* if every uncountable subset of X^2 that is disjoint from the diagonal has an uncountable subset whose closure is disjoint from the diagonal.

Small diagonals

A space, X, has a *small diagonal* if every uncountable subset of X^2 that is disjoint from the diagonal has an uncountable subset whose closure is disjoint from the diagonal.

A well-known question: if X is compact Hausdorff with a small diagonal (a csD space) then is X metrizable?

Delft University of Technology

Small diagonals

Answer: yes if . . .

Small diagonals

Answer: yes if . . .

- CH holds

Small diagonals

Answer: yes if ...

- CH holds
- PFA holds

Small diagonals

Answer: yes if . . .

- CH holds
- PFA holds
- in any world obtained from a CH world by property K forcing

Small diagonals

Answer: yes if . . .

- CH holds
- PFA holds
- in any world obtained from a CH world by property K forcing
- the weight is at most $\aleph_1$

Small diagonals

Answer: yes if . . .

- CH holds
- PFA holds
- in any world obtained from a CH world by property K forcing
- the weight is at most $\aleph_1$
- the space is hereditarily Lindelöf

Small diagonals

Answer: yes if . . .

- CH holds
- PFA holds
- in any world obtained from a CH world by property K forcing
- the weight is at most $\aleph_1$
- the space is hereditarily Lindelöf
- . . .

Small diagonals

Answer: yes if ...

- CH holds
- PFA holds
- in any world obtained from a CH world by property K forcing
- the weight is at most $\aleph_1$
- the space is hereditarily Lindelöf
- ...

the list seems endless.

Delft University of Technology

Why invisible?

There are no illuminating examples of csD spaces.

Why invisible?

There are no illuminating examples of csD spaces.

Because the question is still open (metrizable is not very illuminating).

Why invisible?

There are no illuminating examples of csD spaces.

Because the question is still open (metrizable is not very illuminating).

We have, I believe, squeezed the last drop out of ‘hereditarily Lindelöf’

Why invisible?

There are no illuminating examples of csD spaces.

Because the question is still open (metrizable is not very illuminating).

We have, I believe, squeezed the last drop out of ‘hereditarily Lindelöf’: only one $\aleph_1$-sized subspace arising from one suitable elementary substructure needs to be Lindelöf for a csD space to be metrizable.

Delft University of Technology

Perch and scruple

perch: a unit of length equal to $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet

Perch and scruple

perch: a unit of length equal to $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet

The walk from the Miller Inn to Malott is little over one kilo-perch.

Perch and scruple

perch: a unit of length equal to $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet

The walk from the Miller Inn to Malott is little over one kilo-perch.

scruple: a unit of apothecaries' weight equal to 20 grains or $\frac{1}{3}$ dram ($\frac{1}{24}$ of an ounce)

Outline

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.

Delft University of Technology

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.
So $U(\omega_1)$ seems to be at least as big as ω^*.

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.
So $U(\omega_1)$ seems to be at least as big as ω^*.

Surprisingly: after c.c.c. forcing over a model of $2^{\aleph_1} = \aleph_2$ the cellularity of this space is (still) $\aleph_2$

Delft University of Technology

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.
So $U(\omega_1)$ seems to be at least as big as ω^*.

Surprisingly: after c.c.c. forcing over a model of $2^{\aleph_1} = \aleph_2$ the cellularity of this space is (still) $\aleph_2$, which can be way less than $\mathfrak{c}$.

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.
So $U(\omega_1)$ seems to be at least as big as ω^*.

Surprisingly: after c.c.c. forcing over a model of $2^{\aleph_1} = \aleph_2$ the cellularity of this space is (still) $\aleph_2$, which can be way less than $\mathfrak{c}$.

In the special case of Cohen reals the density is (still) $\aleph_2$.

$U(\omega_1)$

The space $U(\omega_1)$ of uniform ultrafilters on ω_1 has cardinality $2^{\aleph_1}$, which is at least $\mathfrak{c}$.
So $U(\omega_1)$ seems to be at least as big as ω^*.

Surprisingly: after c.c.c. forcing over a model of $2^{\aleph_1} = \aleph_2$ the cellularity of this space is (still) $\aleph_2$, which can be way less than $\mathfrak{c}$.

In the special case of Cohen reals the density is (still) $\aleph_2$.
In fact, it is co-absolute with $\omega_2^\omega \times 2^\kappa$, where κ is the number of Cohen reals added.

Delft University of Technology

$U(\omega_1)$

On the other hand: an ω_3-long finite-support iteration of $\mathrm{Fn}(\omega, 2) * \dot{\mathbb{S}}$ over a model of GCH gives a model where $U(\omega_1)$ is no longer co-absolute with any product of discrete spaces.

Delft University of Technology

$U(\omega_1)$

On the other hand: an ω_3-long finite-support iteration of $\mathrm{Fn}(\omega, 2) * \dot{\mathbb{S}}$ over a model of GCH gives a model where $U(\omega_1)$ is no longer co-absolute with any product of discrete spaces.
($\dot{\mathbb{S}}$ is a Cohen-real-name for a Souslin tree.)

$U(\omega_1)$

On the other hand: an ω_3-long finite-support iteration of $\mathrm{Fn}(\omega, 2) * \dot{\mathbb{S}}$ over a model of GCH gives a model where $U(\omega_1)$ is no longer co-absolute with any product of discrete spaces.
($\dot{\mathbb{S}}$ is a Cohen-real-name for a Souslin tree.)

The cellularity of $U(\omega_1)$ is $\aleph_2$

$U(\omega_1)$

On the other hand: an ω_3-long finite-support iteration of $\mathrm{Fn}(\omega, 2) * \dot{\mathbb{S}}$ over a model of GCH gives a model where $U(\omega_1)$ is no longer co-absolute with any product of discrete spaces.
($\dot{\mathbb{S}}$ is a Cohen-real-name for a Souslin tree.)

The cellularity of $U(\omega_1)$ is $\aleph_2$,
its π-weight is $\aleph_3$

$U(\omega_1)$

On the other hand: an ω_3-long finite-support iteration of $\mathrm{Fn}(\omega, 2) * \dot{\mathbb{S}}$ over a model of GCH gives a model where $U(\omega_1)$ is no longer co-absolute with any product of discrete spaces.
($\dot{\mathbb{S}}$ is a Cohen-real-name for a Souslin tree.)

The cellularity of $U(\omega_1)$ is $\aleph_2$,
its π-weight is $\aleph_3$,
we forgot to (or could not) determine its density.

Outline

Delft University of Technology

Alan

harmony (probably Celtic)

Alan

harmony (probably Celtic)

Delft University of Technology

I guess you all will want me to . . .

Say no more!

Delft University of Technology