Stimulating creative problem solving in mathematics

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Tom Poes, devise a cunning plan!

Creative?

To begin:

I do not believe in creativity (in Mathematics)
without a solid base.

Yet . . .

Implicitly we ask students to be creative *all the time*.

Every problem/exercise demands (some) creativity, if only when choosing a solution method.

Yet . . .

In our first year there are three courses that ‘really’ need creativity.

1. Kaleidoscope
2. Modeling 1A
3. Modeling 1B

Today I will talk about Kaleidoscope.

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Modeling

A bit about the Modeling courses.

Here students are thrown, in pairs (1A) or larger groups (1B), in the deep end (the shallow end actually) with a problem from the 'real' world.

These problems do not have a right/wrong answer.

At the end of 1B: poster presentations.

Kaleidoscope

This course is taught in the first quarter and has, among others, lectures on the following diverse subjects:

1. Graphs
2. Optimisation
3. Differential equations
4. Complex numbers
5. Counting (me)
6. Probability/Statistics

Not necessarily in that order.

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Goals?

Goals of Kaleidoscope.

- See some of the diversity of Mathematics
- See other ways of thinking/arguing
- Show that not everything goes along well-marked paths
- Collaboration

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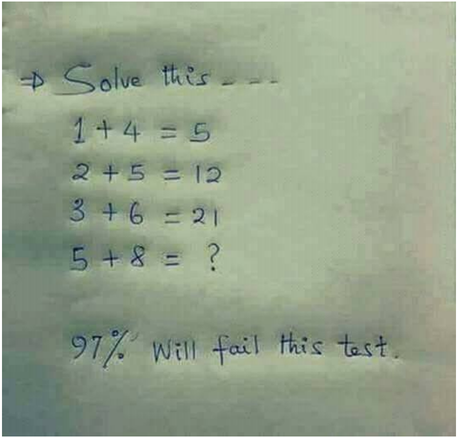
What we do

Six times:

- Lecture (each subject by a different person)
- Exercise class
- Mentorship/tutorial

Mentorship: one staff member gets about 10–12 students
Work a number of assignments in weekly changing subgroups.
This work has to be handed in.

A ‘mathematical’ problem



Another viral problem

Which is larger

$$1000^{1001} \qquad \text{of} \qquad 1001^{1000}$$

Find at least three solutions.

Divisions into groups

Question for me

Find a way to divide the mentor group six times into three groups, in such a way that students share a group as few times as possible.

Shaking hands (Graphs)

The question

At a birthday party there are 100 people present. These people shake hands with some (and in one case with all) of the others. After counting and reordering, it turns out that among the first 99 people present, the first person shook hands with one other person, the second person shook hands with two others, the third person shook hands with 3 people, et cetera (so the 99th person shook hands with 99 other people). Of the last person, mister X, this number (say x) was not counted. Determine the number of people X shook hands with (so find x).

Shaking hands (Graphs)

The hint

Hint: First replace 100 by 6 (or number of similar size). So consider the situation in which 6 people shake hands with 1, 2, 3, 4, 5, and x others, respectively.

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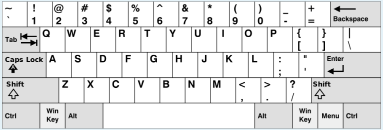
Passwords

The question

Max needs to make a password for his TU Delft Netid. It needs to contain lower case letters, upper case letters, digits and special symbols.



Max decides on a twelve-character password with three characters from each category; these can occur in any position. So `G00DPa$$w9r&` would be a good password. How many password can Max make?

Blind dates (Counting)

From the `wisfaq` website:
http://www.wisfaq.nl/show3archive.asp?id=75609&j=2015

The question

There is a blind date session with 2 men and 3 women. Each man may choose one women and each women may choose one man. There is a match when a man and a women choose each other (man A chooses woman B / woman B chooses man A). Possible outcomes per set of choices: no match, 1 match, or 2 matches.

Blind dates (Counting)

The question (continued)

When I work out all combinations for 2 men and 3 women in a spreadsheet, then I find 72 possible combinations ($2^3 \times 3^2 = 8 \times 9 = 72$), with 12 times 2 matches, 48 times 1 match and 12 times no match.

It is undoable to work out, in a spreadsheet, the possibilities for 4 men and 3 women.

There should/must be a general formula for this.

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Blind dates (Counting)

The question (continued)

When I have a men and b women, then the number of possible matches is:

- $a+1$, when $a < b$
- $b+1$, when $b < a$

(2 me and 3 women gives $2+1=3$ possible outcomes: $0\times$, $1\times$ or $2\times$ a match).
The number of possible combinations is $a^b \times b^a$.

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Blind dates (Counting)

The question (continued)

What then is the formula to calculate how often every match occurs
(with 2 men and 3 women there are three possible outcomes for 72 possible combinations, where the three possible outcomes occur 12, 48, and 12 times respectively).

Blind dates (Counting)

I used this problem in the Counting chapter.

We asked for a formula for the number of possibilities with 0 matches.

First try to calculate the poser's numbers for $a = 2$ and $b = 3$.

Then try his 'undoable' case: $a = 3$ and $b = 4$.