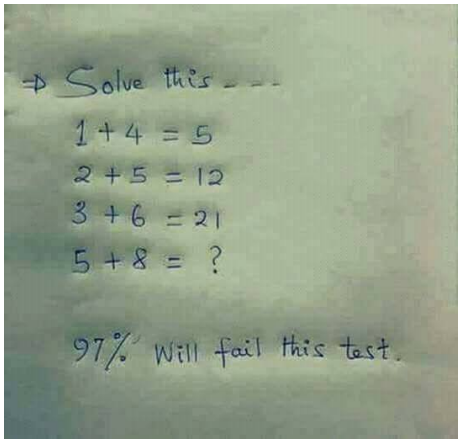
Question 1



Question 2

Which is larger

$$1000^{1001} \qquad \text{of} \qquad 1001^{1000}$$

Find at least three different ways of deciding this.

Question 3

Find a way to divide the mentor group six times into three groups, in such a way that students share a group as few times as possible.

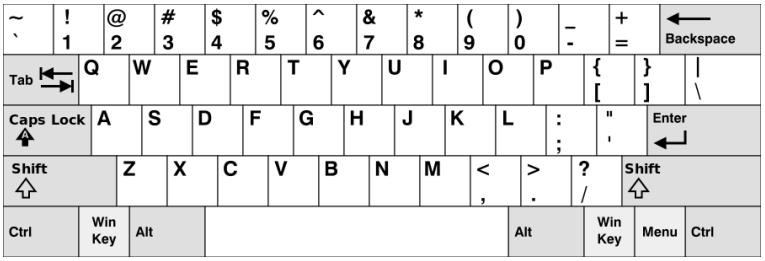
Question 4

At a birthday party there are 100 people present. These people shake hands with some (and in one case with all) of the others. After counting and reordering, it turns out that among the first 99 people present, the first person shook hands with one other person, the second person shook hands with two others, the third person shook hands with 3 people, et cetera (so the 99th person shook hands with 99 other people). Of the last person, mister X, this number (say x) was not counted. Determine the number of people X shook hands with (so find x).

Hint. First replace 100 by 6 (or number of similar size). So consider the situation in which 6 people shake hands with 1, 2, 3, 4, 5, and x others, respectively.

Question 5

Max needs to make a password for his TU Delft Netid. It needs to contain lower case letters, upper case letters, digits and special symbols.



Max decides on a twelve-character password with three characters from each category; these can occur in any position. So `G00DPa$$w9r&` would be a good password. How many password can Max make?

Question 6

From the `wisfaq` website: `http://www.wisfaq.nl/show3archive.asp?id=75609&j=2015`

There is a blind date session with 2 men and 3 women. Each man may choose one women and each women may choose one man. There is a match when a man and a women choose each other (man A chooses woman B / woman B chooses man A).

Possible outcomes per set of choices: no match, 1 match, or 2 matches.

When I work out all combinations for 2 men and 3 women in a spreadsheet, then I find 72 possible combinations ($2^3 \times 3^2 = 8 \times 9 = 72$), with 12 times 2 matches, 48 times 1 match and 12 times no match.

It is undoable to work out, in a spreadsheet, the possibilities for 4 men and 3 women.

There should/must be a general formula for this.

When I have a men and b women, then the number of possible matches is:

- $a + 1$, when $a < b$
- $b + 1$, when $b < a$

(2 me and 3 women gives $2 + 1 = 3$ possible outcomes: $0\times$, $1\times$ or $2\times$ a match).

The number of possible combinations is $a^b \times b^a$.

What then is the formula to calculate how often every match occurs

(with 2 men and 3 women there are three possible outcomes for 72 possible combinations, where the three possible outcomes occur 12, 48, and 12 times respectively).

I used this problem in the Counting chapter. The problem asked to find a formula for the number of possibilities with 0 matches.

Hint. First try to calculate the poser's numbers for $a = 2$ and $b = 3$.

Then try his 'undoable' case: $a = 3$ and $b = 4$.