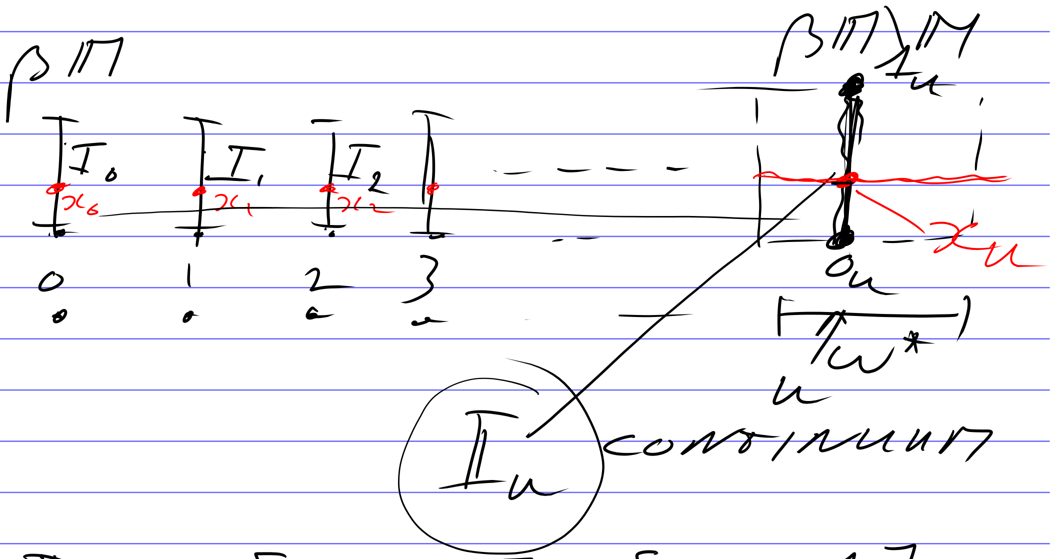
TWO MODEL

1 CUT POINTS IN $\beta\mathbb{R}\setminus\mathbb{R}$
THE SUBCONTINUA THEREOF

$\mathbb{M} = \omega \times [0,1]$

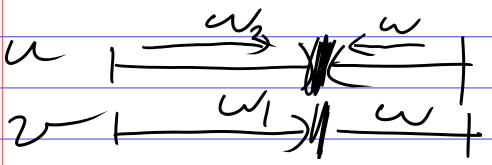


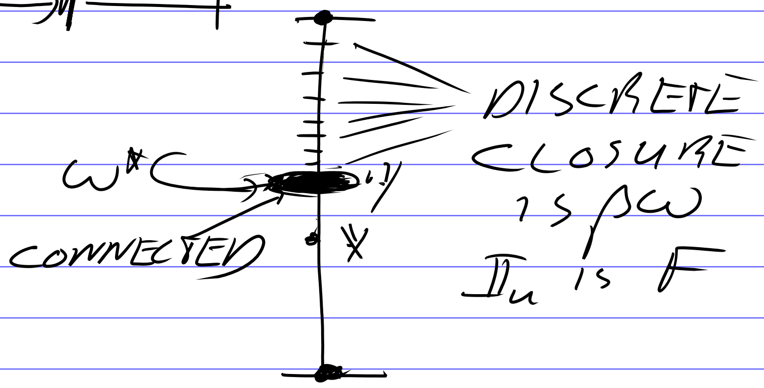

$\mathbb{I}_u = [0_u, x_u] \cup [x_u, 1_u]$

$\{x_u\} = [0_u, x_u] \cap [x_u, 1_u]$

ALL THOSE CUT POINTS FORM $[0,1]^\omega / u$

CH: ALL $\mathbb{I}_u$ HOMEOMORPHIC




ANY OTHER CUT POINTS?

$MA_{ctble}$ SOME $u$
SOME $x$ CUT POINT
$x$ REMOTE FROM $\mathbb{M}$

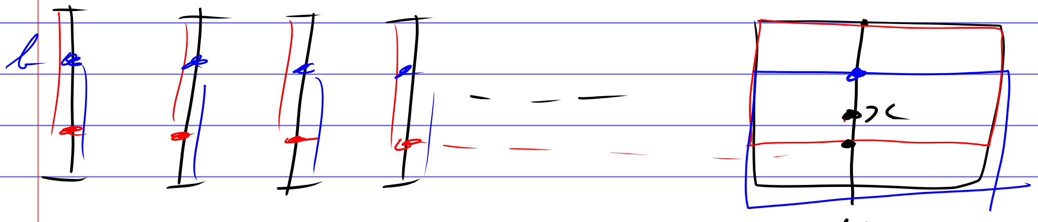
$x \notin \overline{F}$ $F$ NOWHERE DENSE IN $\mathbb{M}$

ZHU: IN LAVER MODEL
NO REMOTE CUT POINTS

A NON-TRIVIAL CUT POINT IS A FAR POINT

$x \notin D$ IF $D$ IS CLOSED DISCRETE IN $M$.

$D \cap (\{n\}\times[0,1])$ IS FINITE



$A_x = \{ \langle a_n \rangle_n : x \in \overline{\bigcup_n \{n\}\times[a_n,1]} \}$

$B_x = \{ \langle b_n \rangle_n : x \in \overline{\bigcup_n \{n\}\times[0,b_n]} \}$

$x$ IS A TRIVIAL CUTPOINT

$x = x_u \longleftarrow \langle x_n \rangle_n$

$\langle x_n \rangle_n \in A_x \cap B_x$

$x$ NOT A TRIV. CUT P.

$\iff$

$A_x \cap B_x = \emptyset$

$x$ IS A CUT POINT IFF

- FOR EVERY $f : \omega \to \omega$ THERE ARE $a \in A_x, b \in B_x$ ST $\{ n : b_n - a_n < 2^{-f(n)} \} \in u$

- FOR EVERY $f : \omega \to \mathbb{N}$ THERE IS A $g : \omega \to \omega$ WITH $g(n) < f(n)$ AND

$$\bigcup_n \{n\} \times \left[ \frac{g(n)}{f(n)}, \frac{g(n)+1}{f(n)} \right] \in x$$

$F(g,f)$

IF $D$ IS A CLOSED DISCRETE
THE WE HAVE $f: \omega \to \mathbb{N}$
IF $\langle n, d\rangle, \langle n, e\rangle \in D$ $d \neq e$
THEN $\frac{B}{f(n)} < |d-e|$

TAKE THE $g$
THEN $F(g,f) \in x$

$|F(g,f) \cap D \cap \{n\} \times [0,1]| \leq 1$

INDUCTION:
IF $D$ IS CLOSED SCATTERED OF FINITE RANK
THEN $x \notin D$

CH: THERE IS A CLOSED SET $E$ SCATTERED OF RANK $\omega$
SUCH THAT $E$ ~~HAS~~ CONTAINS A NON-TRIV. CUT POINT IN EVERY $V_u$

THE BOREL CONJECTURE

$X \subseteq \mathbb{R}$ HAS STRONG MEASURE ZERO:

IF $\langle \varepsilon_n \rangle_n$ IS A SEQ OF POSITIVE REALS THERE IS A SEQ. OF INTERVALS $\langle I_n \rangle_n$ ST
- $\lambda(I_n) \leq \varepsilon_n$
- $X \subseteq \bigcup_n I_n$

BOREL:
STRONG MEAS ZERO $\Longrightarrow$ COUNTABLE

CH $\to$ LUSIN SET — HAS SMZ
LUSIN: — UNCOUNTABLE
- INTERSECTS EVERY NWD SET IN A COUNTABLE SET.

LAVER 1970'S
CON(BOREL CONJECTURE)

BAUMGARTNER'S TRANSL.

WORKS IN CANTOR SET
$^{\omega}2$

$s \in {}^{<\omega}2$ $\quad [s] = \{x : s \subseteq x\}$

MEASURE $2^{-|s|}$

GIVES A MEASURE ON $^{\omega}2$

$X \subseteq {}^{\omega}2$ HAS SMZ IFF
FOR EVERY $f : \omega \to \omega$
THERE IS $g : \omega \to {}^{<\omega}2$
WITH: $-$ $g(n) \in {}^{f(n)}2$

$$X \subseteq \bigcup \{[g(n)] : n \in \omega\}$$

WE SHOULD PROVE

(1) IF $x$ IS A FAR POINT IN $\mathbb{N}^*$
THEN THERE IS
A FUNCTION $f : \omega \to \mathbb{N}$
SUCH THAT FOR NO
$g : \omega \to \omega$ DOES
$g(n) < f(n)$ $F(g, f)$ BELONG TO $x$

(2) IF $X \subseteq {}^{\omega}2$ IS UNCTBLE
THEN THERE IS $f : \omega \to \omega$
SUCH THAT FOR NO
$g : \omega \to {}^{<\omega}2$ WITH $g(n) \in {}^{f(n)}2$
DOES $X \subseteq \bigcup_n [g(n)]$

$\mathbb{L}$

ITERATE $\omega_2$ TIMES
COUNTABLE SUPPORTS

$\mathbb{P}_0$ -- $\mathbb{P}_\alpha$ —— $\mathbb{P}_{\omega_2}$

$x$ FAR POINT IN $V[G_{\omega_2}]$

$X$ UNCOUNTABLE IN SIDE $\aleph_1$

$\dot{x}$ $\dot{X}$

$\alpha \Vdash_\alpha \dot{x} \upharpoonright \alpha$ IS FAR

$X \in V[G_\alpha]$

$V[G_\alpha]$ — $x \upharpoonright \alpha$ FAR
$X$ UNCTBLE $\aleph_1$

CH

REST OF THE ITERATION
IS JUST THE ITERATION
PERFORMED IN $V[G_\alpha]$

WLOG: $x$ AND $\underline{X}$
ARE IN $V$

$x$ IS A SHADOW OF
ITS FUTURE SELF

FOR EVERY $g$
THERE IS $F$ IN $x$ IN $V$
WITH $F \cap F(g, f) = \emptyset$

FOR EVERY $g$
$X \notin U_n[g(n)]$

$\mathbb{L} = \{ T \subseteq {}^{<\omega}\omega : T$ IS A TREE SUCH THAT THERE $s_T \in T$

ST IF $t \in T$

THEN $t \subseteq s_T$

OR $s_T \subseteq t$

$s_T$

IF $t \in T$ AND

$s_T \subseteq t$

THEN

$\{n : t^\frown n \in T\}$

IS INFINITE $\}$

ORDER $S \underset{\subseteq}{\leq} T$

$[T] = \{ x \in {}^{\omega}\omega : \forall n \; x \restriction n \in T \}$

$G$ GENERIC ON $\mathbb{L}$

THEN $f = \bigcup \{ s_T : T \in G \}$

$\{f\} = \bigcap \{ [T] : T \in G \}$

THIS IS THE $f$