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Brouwer and cardinalities. (English. English summary)

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This is a pleasant first-person account of the author's reading of L. E. J. Brouwer's contribution to the IVth International Congress of Mathematicians (of 1908). As Hart explains, Brouwer's note, standing at barely more than two pages, makes for difficult reading today: standards of clarity and rigor at the time were not what they are now, we no longer have access to common understandings of their time, and Brouwer was still developing his philosophy and mathematics and so could not be expected to have had a mature and thorough framework for his thoughts. The author's goal is to explain Brouwer's paper on its own terms, and to give a modern view of it.

Brouwer's topic was possible infinite cardinalities. He concludes that there are three: the countable; the countably unfinished, which is given by a method that from every counting produces an element not so listed, which was not taken as evidence of uncountability; and the continuum, which was taken as something finished, but not a set. Hart gives a sympathetic explication of this content, while not shying away from a critical exposure of what he finds wanting in Brouwer's writing. After explaining and criticizing Brouwer's ICM publication in isolation, Hart then brings in other contemporaneous work, such as Zentralblatt's review of Brouwer and some germane work of Cantor, and concludes that even at the time Brouwer's meaning was unclear. In the final section, and throughout the paper, Hart gives a modern view of what Brouwer meant and could have meant.

The current paper concludes with a Brouwerian anecdote warning us that Hart might have misread everything. To which this reviewer reacts by thinking that, while a reader should cut the writer some slack, it really is the latter's responsibility to make sure that they are not misunderstood.

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[References]

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.