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Brouwer and cardinalities. (English) Zbl 1406.03008
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This article presents a pertinent and sharp analysis of a brief paper of *L. E. J. Brouwer* with the title “Die möglichen Mächtigkeiten” [Rom. 4. Math. Kongr. 3, 569–571 (1909; [JFM 40.0099.04](#))] the subject of which is the foundation of mathematics. The author of the paper under review conveys without ambiguity the domain, the perspective and the limits of his research: (1) “I should emphasize that I read the paper first as it is, without consulting any background material” (2) “My reading is influenced by my own mathematical experience: I fully embrace modern set theory, excluded middle, axiom of choice, and all.” I think that such a lucidity enhances the epistemological scope of the present work. In the following lines, I shall try to clarify this point. First of all, I am going to follow the footsteps, so to say, of a theme which plays a most relevant part: (i) “So I started to read the paper more closely and I found it harder to understand than expected; the definitions and arguments are not always as concrete as I would have like them to be...many notions had a meaning that was tacitly assume to be the same to every reader” (p. 1556) (ii) This was hard to make clear sense of “primordial intuition”, “Zweieinigkeit”, “The first”, “The second”, “the continuous”...seem to lack concrete meaning...one can argue that the paper is a product of its time and that in this context these terms would make sense to a reader back in the day” (p. 1579) (iii) “it has been suggested to me that I should take the time of writing into consideration and look on Brouwer as a nineteenth-century mathematician. And it must be said that many papers from that era tacitly assume that many things are understood by everyone to mean the same thing...” (p. 1561). In these passages, the author points to a meaning which is “tacitly assumed”. And here lies, I think, the gist of the matter. *M. Polanyi* [The tacit dimension. Chicago, IL: The University of Chicago Press (1966), p. 4 and p. 9] put forward, many years from now, a rather rich and deep idea: “I shall reconsider human knowledge by starting from the fact that we can know more than we can tell...Here we see the basic structure of tacit knowing. It always involves two things or two kinds of things. We may call them the two terms of tacit knowing...”. This is not the place to treat with due care the conceptual horizon of “tacit knowledge”. However, it may be useful to illustrate “the two kinds of things” mentioned by Polanyi. “The playing of a game of chess is an entity controlled by principles which rely on the observance of the rules of chess; but the principles controlling the game cannot be derived from the rules of chess” [loc. cit., p. 34]. More recently, *H. Bregger* [in: The growth of mathematical knowledge. Dordrecht: Kluwer Academic Publishers. 221–230 (2000; [Zbl 0951.00515](#))] indicated that “Mathematics as a purely formal system of symbols without a human being possessing the know-how for dealing with symbols is impossible. According to the chemist Polanyi the ideal of a form of knowledge that is strictly explicit is contradictory because without the tacit knowledge all formulas, words, and illustrations would become meaningless”. It seems that it is possible to perceive the heart of the author’s purpose: to read between the lines in order to translate the arguments of Brouwer “to our standard situation”. But the last loop of this line of thought is subtle for the author is deemed to uncover his own tacit knowledge to discover the tacit knowledge of Brouwer. A daring adventure indeed.

Reviewer: [Godofredo T. Iommi Amunategui \(Valparaíso\)](#)

MSC:

- 03-03 History of mathematical logic and foundations
- 03E10 Ordinal and cardinal numbers
- 03E17 Cardinal characteristics of the continuum
- 03F55 Intuitionistic mathematics
- 01A60 History of mathematics in the 20th century
- 01A70 Biographies, obituaries, personalia, bibliographies

Keywords:

[cardinalities](#); [continuum](#); [closed set](#)

Biographic references:

[Brouwer, L. E. J.](#)

Full Text: [DOI](#) [arXiv](#) [Link](#)

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