Problems in Topology II
Edited by E. Pearl, Pages 171–177
S (XX)0000-0

EFIMOV'S PROBLEM

KLAAS PIETER HART

Introduction

In their memoir [1, page 54] Alexandroff and Urysohn asked "*existe-il un espace compact (bicompact) ne contenant aucun point* ($\varkappa$)*?*" and went on to remark "*La resolution affirmative de ce problème nous donnerait une exemple des espaces compacts (bicompacts) d'une nature toute differente de celle des espaces connus jusqu'à présent*". The 'compact' of that memoir is our countably compact, 'bicompact' is '*compact Hausdorff*' and a $\varkappa$*-point* is one that is the limit of a non-trivial convergent sequence. A look through the examples in [1] will reveal a few familiar classics: the ordinal space ω_1 and the corresponding Long line, the double circumference, the Tychonoff plank (in disguise), the lexicographically ordered square, and the Double Arrow space. The geometric nature of the constructions made the introduction of non-trivial convergent sequences practically unavoidable and it turns out that the remark was quite correct as we will see below.

The question was answered by Tychonoff [20] and Čech [4] using the very same space, though their presentations were quite different. Tychonoff took for every $x \in (0,1)$ its binary expansion $0.a_1(x)a_2(x)\dots a_n(x)\dots$ (favouring the one that ends in zeros), thus creating a countable set $\{a_n : n \in \mathbb{N}\}$ of points in the *Tychonoff cube* $[0,1]^{(0,1)}$, whose closure is the required space. Čech developed what we now call the *Čech-Stone compactification*, denoted βX, of completely regular spaces and showed that $\beta\mathbb{N}$, where $\mathbb{N}$ is the discrete space of the natural numbers, has no convergent sequences.

A natural question is whether one has to go to such great lengths to construct a compact Hausdorff space without convergent sequences. This then is Efimov's problem, raised in [10].

Efimov's problem. *Does every infinite compact Hausdorff space contain either a non-trivial convergent sequence or else a copy of* $\beta\mathbb{N}$*?* **397 ?**

It should be noted that Efimov raised his problem not in the context sketched above but as part of a program to determine when Čech-Stone compactifications of discrete spaces were embeddable in certain compact Hausdorff spaces.

Date: Monday 23-04-2007 at 13:52:53 (cest).
2000 *Mathematics Subject Classification.* Primary: 54F12. Secondary: 03C20, 03C35, 54F50.
Key words and phrases. compact Hausdorff space, convergent sequence, $\beta\mathbb{N}$.

For the rest of this note all convergent sequences will be assumed to be non-trivial, so that adjective will not be used. Basic information about $\beta\mathbb{N}$ may be found in [8].

1. Attacking the problem

Efimov's problem may of course be cast in the form of an implication: If a compact Hausdorff space does not contain any convergent sequences must it then contain a copy of $\beta\mathbb{N}$?

Let us consider an infinite compact Hausdorff space X that does not contain any convergent sequences. As any infinite Hausdorff space, it does contain an infinite relatively discrete subspace; we take a countably infinite subset of that subspace and identify it with $\mathbb{N}$. The sequence $\langle n\rangle_n$ does not converge, so we can take two distinct accumulation points, x_0 and x_1, of $\mathbb{N}$. Take neighbourhoods U_0 and U_1 of x_0 and x_1 respectively with disjoint closures and put $A_0 = U_0 \cap \mathbb{N}$ and $A_1 = U_1 \cap \mathbb{N}$. Thus we find that in an infinite compact Hausdorff space without convergent sequences every countably infinite discrete subset has two infinite subsets with disjoint closures. To get a copy of $\beta\mathbb{N}$ one should construct an infinite discrete subset with the property that any two disjoint subsets have disjoint closures. To appreciate how difficult this may be we continue our construction.

We have our two disjoint subsets of $\mathbb{N}$ with disjoint closures. We iterate the procedure above and determine, recursively, a family $\{A_s : s \in {}^{<\omega}2\}$, where ${}^{<\omega}2$ is the binary tree of finite sequences of zeros and ones, that satisfies

- if $s \subseteq t$ then $A_t \subseteq A_s$, and
- $\operatorname{cl} A_{s*0} \cap \operatorname{cl} A_{s*1} = \emptyset$.

Using this family one defines, for every point x in the Cantor set ${}^{\omega}2$, a closed set $F_x = \bigcap_n \operatorname{cl} A_{x\restriction n}$. By construction the (nonempty) closed sets F_x are disjoint and we see that the cardinality of X must be at least $\mathfrak{c}$. In fact, with some care one can arrange matters so that

- $F = \bigcup_x F_x$ is closed, and
- mapping the points of F_x to x gives a continuous map from F onto ${}^{\omega}2$.

Using the Tietze-Urysohn theorem one can employ this map to obtain a continuous surjection from X onto the unit interval $\mathbb{I}$ or even the Hilbert cube ${}^{\omega}\mathbb{I}$. As we will see below what is needed is a continuous map onto the Tychonoff cube ${}^{\mathfrak{c}}\mathbb{I}$; however, naïvely, the Hilbert cube is best possible. Though the construction above can be continued for (at least) ω_1 many steps to show that $\mathbb{N}$ has at least $2^{\aleph_1}$ many accumulation points, the examples below show that it will not necessarily yield a map onto the next cube ${}^{\omega_1}\mathbb{I}$.

To get a copy of $\beta\mathbb{N}$ inside X more is needed, as Efimov himself established in [10] when he characterized the spaces that do contain such a copy. On the one hand the space $\beta\mathbb{N}$ admits a continuous map onto the *Cantor cube* ${}^{\mathfrak{c}}2$ and thence onto the Tychonoff cube ${}^{\mathfrak{c}}\mathbb{I}$; the Tietze-Urysohn theorem may then be applied to produce a continuous map from the ambient space onto this cube. On the other hand assume that X maps onto ${}^{\mathfrak{c}}\mathbb{I}$. Since the cube contains a copy of $\beta\mathbb{N}$, a standard argument produces a closed subset F of X and an irreducible map from F onto $\beta\mathbb{N}$. Because $\beta\mathbb{N}$ is *extremally disconnected* this map is a homeomorphism.

It follows that the following statements about a compact Hausdorff X are equivalent:

(1) X contains a copy of $\beta\mathbb{N}$,
(2) X maps onto ${}^{\mathfrak{c}}\mathbb{I}$,
(3) some closed subset of X maps onto ${}^{\mathfrak{c}}2$, and
(4) there is a *dyadic system* $\{\langle F_{\alpha,0}, F_{\alpha,1}\rangle : \alpha < \mathfrak{c}\}$ of closed sets in X.

The dyadic system satisfies, by definition,

- $F_{\alpha,0} \cap F_{\alpha,1} = \emptyset$ for all α, and
- $\bigcap_{\alpha \in \operatorname{dom} p} F_{\alpha,p(\alpha)} \neq \emptyset$, whenever p is a finite partial function from $\mathfrak{c}$ to 2.

To deduce 4 from 3 simply set $F_{\alpha,i} = f^{\leftarrow}\bigl(\pi_\alpha^{\leftarrow}(i)\bigr)$, where f is the map onto ${}^{\mathfrak{c}}2$ and π_α is the projection onto the αth coordinate. Conversely, this same formula implicitly defines a continuous map from $\bigcap_{\alpha<\mathfrak{c}}(F_{\alpha,0} \cup F_{\alpha,1})$ onto ${}^{\mathfrak{c}}2$.

In [18] Shapirovskiĭ added another condition to this list: there is a closed set F such that $\pi\chi(x, F) \geq \mathfrak{c}$ for all points of F. Here $\pi\chi(x, F)$ is the *π-character* of x (in F): the minimum cardinality of a family $\mathcal{U}$ of non-empty open sets such that every neighbourhood of x contains an elements of $\mathcal{U}$ (the elements of $\mathcal{U}$ need not be neighbourhoods of x).

2. Counterexamples

There are several consistent counterexamples to Efimov's problem. This of course precludes an unqualified positive answer and leaves us with two possibilities: a real, ZFC, counterexample or the consistency of a positive answer.

Here is a list of the better-known counterexamples.

(1) For every natural number n there is a compact Hausdorff space X_n with the property that every infinite closed subset has covering dimension n. As both the convergent sequence and $\beta\mathbb{N}$ are zero-dimensional neither can be a subspace of X_n. This example was constructed by Fedorčuk in [11] using the Continuum Hypothesis (CH).
(2) Another example, this time with the aid of $\diamondsuit$, was constructed by Fedorčuk in [12]. The space is a compact S-space of size $2^{\mathfrak{c}}$ without convergent sequences. As $\beta\mathbb{N}$ is not hereditarily separable it cannot be embedded into this space.
(3) Yet another counterexample was constructed by Fedorčuk in [13] using a principle he called the Partition Hypothesis. In present day terms this is the conjunction of $\mathfrak{s} = \aleph_1$ and $2^{\aleph_0} = 2^{\aleph_1}$. Here $\mathfrak{s}$ is the *splitting number*, the minimum cardinality of a *splitting family*, that is, a family $\mathcal{S}$ of subsets of $\mathbb{N}$ such that for every infinite subset A of $\mathbb{N}$ there is $S \in \mathcal{S}$ such that both $A \cap S$ and $A \setminus S$ are infinite. Fedorčuk's principle holds in the Cohen model. The title of [13] makes it completely clear why this is a counterexample to Efimov's question: no convergent sequences and the space is simply too small to contain $\beta\mathbb{N}$.

(4) In [6] Dow weakened Fedorčuk's hypothesis substantially, at the cost of a more elaborate construction, to the conjunction of $\mathrm{cf}\bigl([\mathfrak{s}]^{\aleph_0},\subset\bigr)=\mathfrak{s}$ and $2^{\mathfrak{s}}<2^{\mathfrak{c}}$. The former equality says that there are $\mathfrak{s}$ many countable subsets of $\mathfrak{s}$ so that each countable subset of $\mathfrak{s}$ is contained in one of them.

All four counterexamples arise as limits of suitable inverse systems, where at each stage some or all convergent sequences are dealt with. In the first two constructions the CH allows one to do some (clever) bookkeeping so that every potential convergent sequence in the limit is dealt with at some intermediate stage. In the third and fourth example the inverse system is ω_1 long but at every stage all convergent sequences in the space constructed that far are dealt with; the cofinality assumption on $\mathfrak{s}$ enables one to do this by splitting just $\mathfrak{s}$ objects. The final space then has at most $2^{\mathfrak{s}}$ points, so that the power assumption prevents $\beta\mathbb{N}$ from being a subspace.

A simpler version of the third space was given by van Douwen and Fleissner in [5] using $2^{\aleph_0}=2^{\aleph_1}$ plus a version of $\mathfrak{s}=\aleph_1$ for the Cantor set ${}^{\omega}2$: there should be a family $\{U_\alpha:\alpha<\omega_1\}$ of open sets such that for every convergent sequence s there is an α for which $s\cap U_\alpha$ and $s\setminus\mathrm{cl}\,U_\alpha$ are infinite. This example is indeed simpler than the others: after copying the sets U_α to each cube ${}^{\beta}2$ (where $\omega\le\beta<\omega_1$) one can simply write down a formula for the example, as a subspace of the Cantor cube ${}^{\omega_1}2$. Indeed, choose, for each $\beta\ge\omega$, a homeomorphism $h_\beta:{}^{\omega}2\to{}^{\beta}2$ and, for all α, put $U_{\beta,\alpha}=h_\beta[U_\alpha]$. Furthermore let $b:\omega_1\times\omega_1\to\omega_1$ be a bijection with the property that $b(\alpha,\beta)=\gamma$ implies $\beta\le\gamma$. Now the space X is the subspace of ${}^{\omega_1}2$ consisting of those points x that satisfy

$$x\bigl(b(\alpha,\beta)\bigr)=0 \text{ implies } x\restriction\beta\in\mathrm{cl}\,U_{\beta,\alpha}$$

and

$$x\bigl(b(\alpha,\beta)\bigr)=1 \text{ implies } x\restriction\beta\in\mathrm{cl}\,V_{\beta,\alpha}$$

where $V_{\beta,\alpha}={}^{\beta}2\setminus\mathrm{cl}\,U_{\beta,\alpha}$.

3. Is there still a problem?

The condition $\mathrm{cf}\bigl([\mathfrak{s}]^{\aleph_0},\subset\bigr)=\mathfrak{s}$ in used in Dow's example is quite weak; indeed, if it were false an inner model with a measurable cardinal would have to exist. This is explained in [16]: if there is any cardinal κ of uncountable cofinality for which $\mathrm{cf}\bigl([\kappa]^{\aleph_0},\subset\bigr)>\kappa$ then the *Covering Lemma* fails badly: not just for L but for any inner model that satisfies the Generalized Continuum Hypothesis.

One might therefore be tempted to conclude that Efimov's problem is all but solved, especially in the absence of large cardinals. However, that completely disregards the necessary inequality $2^{\mathfrak{s}}<2^{\mathfrak{c}}$; without it the guarantee that the example does not contain $\beta\mathbb{N}$ is gone. We are thus lead to consider situations where $2^{\mathfrak{s}}=2^{\mathfrak{c}}$, or even $\mathfrak{s}=\mathfrak{c}$. The best-known of these is of course when Martin's Axiom (MA) holds and, indeed, it is not (yet) known what the effect of $\mathsf{MA}+\neg\mathsf{CH}$ (or even PFA) is on Efimov's problem.

? 398 **Question 1.** *Does* $\mathsf{MA}+\neg\mathsf{CH}$ (*or* PFA) *imply that a compact Hausdorff space without convergent sequences contains a copy of* $\beta\mathbb{N}$*?*

As noted above, this is equivalent to asking whether such a space admits a continuous map onto ${}^{\mathfrak{c}}\mathbb{I}$. Also, as shown below, under MA every countable and discrete subset of a compact Hausdorff space without convergent sequences has $2^{\mathfrak{c}}$ accumulation points. As such a space does admit a continuous map onto ${}^{\omega}\mathbb{I}$ and considering the adage "MA makes cardinals below $\mathfrak{c}$ countable", it may be worthwhile to investigate the following weaker question first.

Question 2. *Does* MA $+ \neg$CH (*or* PFA) *imply that a compact Hausdorff space without convergent sequences maps onto* ${}^{\omega_1}\mathbb{I}$ *or even onto each cube* ${}^{\kappa}\mathbb{I}$ *for* $\kappa < \mathfrak{c}$? **399 ?**

In case Question 1 has a positive answer it becomes of interest how much of MA is actually needed. The equalities $\mathfrak{s} = \mathfrak{c}$ and $\mathfrak{t} = \mathfrak{c}$ seem to suggest themselves as possible candidates; the former by the rôle of $2^{\mathfrak{s}} < 2^{\mathfrak{c}}$ in the examples of Fedorčuk and Dow and the latter by the fact, shown below, that a countable discrete set in a compact Hausdorff space without convergent sequences has at least $2^{\mathfrak{t}}$ accumulation points. The meaning of $\mathfrak{t}$ will be explained below.

In the CH-type constructions mentioned in Section 2, where one deals with one convergent sequence at a time, the preferred thing to do is to blow up the limit to a larger set, every point of which will be an accumulation point of the sequence. The most frugal thing to do would be to split the limit into just two points. This is called a simple extension and an inverse limit construction where at each step one performs a simple extension never leads to a space that can be mapped onto ${}^{\omega_1}\mathbb{I}$ *unless* the initial space in the system already does so. In Boolean algebraic form this result is due to Koppelberg [17]; in [6] one finds a topological proof and the following question. The definition of 'simple extension' in [6] does not mention the two-point limitation but it is used in the proof.

Question 3. *Is it consistent that every such simple inverse limit contains a convergent sequence? Does* PFA *imply this?* **400 ?**

The jump from a convergent sequence to $\beta\mathbb{N}$ is a large one. The construction in Section 1 suggests that the following question, raised by Hušek in [14], may have a positive answer.

Question 4. *Does every compact Hausdorff space contain either a convergent sequence of length* ω *or one of length* ω_1? **401 ?**

A (non-trivial) convergent sequence of length α in a space X, where α is some (limit) ordinal is an injective map $x : \alpha + 1 \to X$ such that every neighbourhood of $x(\alpha)$ contains $\{x(\gamma) : \beta \leq \gamma < \alpha\}$ for some $\beta < \alpha$.

This question is intimately related to Efimov's problem: it was shown in [3] (and announced in [19]) that $\beta\mathbb{N}$ contains a 'truly' non-trivial convergent sequence of length ω_1: there is a convergent sequence $x : \omega_1 + 1 \to \beta\mathbb{N}$ such that $x(\omega_1) \notin \mathrm{cl}\{x(\gamma) : \gamma < \beta\}$ for all $\beta < \omega_1$.

Hušek's question has an affirmative answer under CH, see [14]; Fedorčuk's S-space is a counterexample to the stronger version, with a 'truly' non-trivial ω_1-sequence.

4. Larger cardinals

In [10] Efimov considered the general problem of characterizing when a space contains a copy of $\beta\kappa$, where κ is any infinite cardinal with the discrete topology. The characterization of embeddability of $\beta\mathbb{N}$ given by Efimov as discussed in Section 1 remains valid in the general situation, as does Shapirovskiĭ's characterization of being able to map a space onto a Tychonoff cube of a given weight.

Many generalizations of Efimov's problem suggest themselves but they will never be as succinct as the original question. Given an uncountable cardinal κ an audacious question would be: *Does every compact Hausdorff space contain either the Alexandroff (one-point) compactification $\alpha\kappa$ of κ or a copy of $\beta\kappa$?*

This would also be a foolish question: an arbitrary compact Hausdorff space need not contain a relatively discrete subset of cardinality κ. A better question would therefore be

? 402 **Question 5.** *Does every large enough compact Hausdorff space contain either the Alexandroff compactification $\alpha\kappa$ of κ or else a copy of $\beta\kappa$?*

This of course begs the question what 'large enough' should mean. Therefore one should first investigate for what class of spaces Question 5 actually makes sense. The answer will have to involve some kind of *structural* description of 'large enough' because for every cardinal κ the ordinal space $\kappa+1$ contains neither $\alpha\omega_1$ nor $\beta\omega_1$, so that size alone does not seem to matter.

Efimov's question is a structural question in disguise: if a compact Hausdorff space does not contain a convergent sequence then can one find a dyadic system of cardinality $\mathfrak{c}$? One may disregard the structural part and concentrate on the cardinality part only to get a weaker version of Efimov's question:

? 403 **Question 6.** *If in a compact Hausdorff space every countable and discrete set has more than one accumulation point must there be such a set with $2^{\mathfrak{c}}$ accumulation points?*

The naïve construction from Section 1 shows that one always gets at least $2^{\aleph_1}$ accumulation points and, naïve though it may be, it does show that the answer to this question is positive under MA: one gets a family $\{F_s : s \in {}^{<\mathfrak{c}}2\}$ of closed sets indexed by the complete complete binary tree of height $\mathfrak{c}$ and such that always $F_{s*0} \cap F_{s*1} = \emptyset$; in this way one obtains a pairwise disjoint family $\{F_x : x \in {}^{\mathfrak{c}}2\}$ of nonempty closed sets, all contained in the derived set of the initial countable and discrete set. To be precise, the construction can be continued all the way up to the cardinal $\mathfrak{t}$, which is, by definition, the minimum cardinal κ for which there is a sequence $\langle A_\alpha : \alpha < \kappa\rangle$ of infinite subsets of $\mathbb{N}$ such that $A_\alpha \subset^* A_\beta$ whenever $\beta < \alpha$ but for which no infinite set A exists with $A \subset^* A_\alpha$ for all α. This shows, as promised above, that the discrete set has at least $2^{\mathfrak{t}}$ accumulation points.

One cannot simply copy Question 6 to larger cardinals: if α is a compact ordinal space and D a (discrete) subset of uncountable size κ then D has κ many accumulation points. However, we can build the partial result on Question 6 into its translation. The strongest version that we get is the following.

Question 7. *Let κ be an infinite cardinal. For what compact Hausdorff spaces X is the following implication valid? If $|D^d| > \kappa$ for all discrete subsets of size κ then $|D^d| \geq 2^{2^\kappa}$ for some discrete subset of size κ.* **? 404**

Here D^d denotes the set of accumulation points of D. Various (weaker) versions of this question can be obtained by inserting $|D^d| \geq \lambda$ in the antecedent and $|D^d| \geq \mu$ into its consequent for cardinals λ and μ that satisfy $\kappa < \lambda < \mu \leq 2^{2^\kappa}$.

A related notion was defined by Arkhangel′skiĭ in [2]: denote by $g(X)$ the supremum of cardinalities of closures of discrete subsets of the space X; Arkhangel′skiĭ asked whether $g(X) = |X|$ for compact Hausdorff spaces. The following question combines this with a sup = max problem.

Question 8. *When does a compact Hausdorff space X have a discrete subset D such that $|\operatorname{cl} D| = |X|$?* **405 ?**

Efimov [9] showed that this is true for dyadic spaces (provided every inaccessible cardinal is strongly inaccessible); in [7] Dow shows that relatively small (cardinality at most $\aleph_\omega$) compact Hausdorff spaces of countable tightness do have such discrete subsets and also gives some consistent examples of compact Hausdorff spaces X of cardinality $\aleph_2$ with $g(X) \leq \aleph_1$; and in [15] Juhász and Szentmiklóssy showed that the GCH implies that *every* compact Hausdorff space of countable tightness has a discrete subset whose closure is as large as the space itself.

Acknowledgement. Thanks to Alan Dow for his help in improving some of the questions raised in this note.

References

[1] P. S. Alexandroff and P. Urysohn, *Mémoire sur les espaces topologiques compacts*, Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam, Eerste Sectie **14** (1929), 93 pp. JFM 55.0960.02

[2] A. V. Arkhangel′skiĭ, *An extremal disconnected bicompactum of weight $\mathfrak{c}$ is inhomogeneous*, Doklady Akademiĩa Nauk USSR **175** (1967), 751–754 (Russian); English transl., Soviet Mathematics. Doklady **8** (1967), 897–900. MR 36 #2122, Zbl. 0153.52202

[3] Bohuslav Balcar, Petr Simon, and Peter Vojtáš, *Refinement properties and extensions of filters in Boolean algebras*, Transactions of the American Mathematical Society **267** (1981), no. 1, 265–283. MR **82k:**06014

[4] Eduard Čech, *On bicompact spaces*, Annals of Mathematics (2) **38** (1937), 823–844. MR 1503374, Zbl. 0017.42803

[5] Eric K. van Douwen and William G. Fleissner, *Definable forcing axiom: an alternative to Martin's Axiom*, Topology and its Applications **35** (1990), 277–289. MR **91k:**03133

[6] Alan Dow, *Efimov spaces and the splitting number*, Topology Proceedings **29** (2005), 105–113. Spring Topology and Dynamical Systems Conference. MR 2182920

[7] ———, *Closures of discrete sets in compact spaces*, Studia Scientiarum Mathematicarum Hungarica **42** (2005), 227–234. MR **2006c:**54004

[8] Alan Dow and Klaas Pieter Hart, *The Čech-Stone Compactifications of $\mathbb{N}$ and $\mathbb{R}$*, Encyclopedia of general topology (Klaas Pieter Hart, Jun-iti Nagata, and Jerry E. Vaughan, eds.), Elsevier Science Publishers B.V., Amsterdam, 2004, pp. 213–217.

[9] B. Efimov, *Subspaces of dyadic bicompacta*, Doklady Akademiĩa Nauk USSR **185** (1969), 987–990 (Russian); English transl., Soviet Mathematics. Doklady **10** (1969), 453–456. MR 39 #3459, Zbl. 0191.21002

[10] ———, *The imbedding of the Stone-Čech compactifications of discrete spaces into bicompacta*, Doklady Akademii͡a Nauk USSR **189** (1969), 244–246 (Russian); English transl., Soviet Mathematics. Doklady **10** (1969), 1391–1394. MR 40 #6505, Zbl. 0204.22704

[11] V. V. Fedorčuk, *A bicompactum whose infinite closed subsets are all n-dimensional*, Matematicheskiĭ Sbornik. Novaya Seriya **96 (138)** (1975), 41–62, 167 (Russian); English transl., Mathematics of the USSR. Sbornik **25** (1976), 37–57. MR 50 #14701, Zbl. 0327.54031

[12] ———, *Completely closed mappings, and the consistency of certain general topology theorems with the axioms of set theory*, Matematicheskiĭ Sbornik. Novaya Seriya **99 (141)** (1976), 1–26 (Russian); English transl., Mathematics of the USSR. Sbornik **28** (1976), 3–33, 135. MR 53 #14379, Zbl. 0371.54045

[13] ———, *A compact space having the cardinality of the continuum with no convergent sequences*, Mathematical Proceedings of the Cambridge Philosophical Society **81** (1977), 177–181. MR 54 #13827, Zbl. 0348.54030

[14] M. Hušek, *Convergence versus character in compact spaces*, Topology, Vol. I (Proceedings of the Fourth Colloquium, Budapest, 1978), Colloquia Mathematica Societatis János Bolyai, vol. 23, North-Holland, Amsterdam, 1980, pp. 647–651. MR **81m:**54010

[15] I. Juhász and Z. Szentmiklóssy, *Discrete subspaces of countably tight compacta*, Annals of Pure and Applied Logic **140** (2006), no. 1-3, 72–74. MR 2224050

[16] Winfried Just, A. R. D. Mathias, Karel Prikry, and Petr Simon, *On the existence of large p-ideals*, The Journal of Symbolic Logic **55** (1990), 457–465. MR **91g:**03094

[17] Sabine Koppelberg, *Minimally generated Boolean algebras*, Order **5** (1989), 393–406. MR **90g:**06022, Zbl. 0676.06019

[18] B. È. Šapirovskiĭ, *Mappings onto Tikhonov cubes*, Uspekhi Matematicheskikh Nauk **35** (1980), 122–130 (Russian); English transl., Russian Mathematical Surveys **35** (1980), 145–156. International Topology Conference (Moscow State University, Moscow, 1979). MR **82d:**54018, Zbl. 0462.54013

[19] Saharon Shelah, *On P-points, beta(omega) and other results in general topology*, Notices of the American Mathematical Society **25** (1978), A365.

[20] A. Tychonoff, *Über einen Funktionenraum*, Mathematische Annalen **111** (1935), 762–766.Zbl. 0012.30802

Faculty of Electrical Engineering, Mathematics, and Computer Science, TU Delft, Postbus 5031, 2600 GA Delft, the Netherlands

E-mail address: K.P.Hart@TUDelft.NL

URL: http://aw.twi.tudelft.nl/~hart