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A SEPARABLE NON-REMAINDER OF $\mathbb{H}$

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Abstract. We prove that there is a compact separable continuum that (consistently) is not a remainder of the real line.

Introduction

Much is known about the continuous images of $\mathbb{N}^*$, the Čech-Stone remainder of the discrete space $\mathbb{N}$. It is almost trivial to prove that every separable compact Hausdorff space is a continuous image of $\mathbb{N}^*$ (we abbreviate this as '$\mathbb{N}^*$-image'). It is a major result of Parovičenko, from [9], that every compact Hausdorff space of weight $\aleph_1$ is an $\mathbb{N}^*$-image, and in [10] Przymusiński used the latter result to prove that all perfectly normal compact spaces are $\mathbb{N}^*$-images. Under the assumption of the Continuum Hypothesis, Parovičenko's result encompasses all three results: a compact Hausdorff space is an $\mathbb{N}^*$-image if and only if it has weight $\mathfrak{c}$ or less.

In [7] the authors formulated and proved a version of Parovičenko's theorem in the class of continua: every continuum of weight $\aleph_1$ is a continuous image of $\mathbb{H}^*$ (an '$\mathbb{H}^*$-image'), the Čech-Stone remainder of the subspace $\mathbb{H} = [0, \infty)$ of the real line. This result built on and extended the corresponding result for metric continua from [1]. Thus the Continuum Hypothesis (CH) allows one to characterize the $\mathbb{H}^*$-images as the continua of weight $\mathfrak{c}$ or less. The paper [7] contains further results on $\mathbb{H}^*$-images that parallel older results about $\mathbb{N}^*$-images: Martin's Axiom (MA) implies that all continua of weight less than $\mathfrak{c}$ are $\mathbb{H}^*$-images, in the Cohen model the long segment of length ω_2 is not an $\mathbb{H}^*$-image, and it is consistent with MA that not every continuum of weight $\mathfrak{c}$ is an $\mathbb{H}^*$-image.

The natural question of whether the 'trivial' result on separable compact spaces has its parallel version for continua has proved harder to answer than expected. We show that in this case the parallelism actually breaks down. There is a well-defined separable continuum K that is not an $\mathbb{H}^*$-image if the Open Colouring Axiom (OCA) is assumed. This also answers a more general question raised by G. D. Faulkner (Question 7.3, [7]): if a continuum is an $\mathbb{N}^*$-image, must it be an $\mathbb{H}^*$-image? Indeed, K *is* separable and hence an $\mathbb{N}^*$-image.

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It is readily seen that $\beta\mathbb{H}$ itself is an $\mathbb{H}^*$-image: by moving back and forth in ever larger sweeps one constructs a map from $\mathbb{H}$ onto itself whose Čech-Stone extension maps $\mathbb{H}^*$ onto $\beta\mathbb{H}$. Indeed the same argument applies to any space that is the union of a connected collection of Peano continua: its Čech-Stone compactification is an $\mathbb{H}^*$-image. Thus, e.g., for every n the space $\beta\mathbb{R}^n$ is an $\mathbb{H}^*$-image. Our example is one step up from these examples: it is the Čech-Stone compactification of a string of $\sin\frac{1}{x}$-curves.

Our result also shows that the proof in [7] cannot be extended beyond $\aleph_1$, as OCA is compossible with Martin's Axiom (MA). The adage that MA makes all cardinals below $\mathfrak{c}$ behave as if they are countable would suggest that the aforementioned proof, an inverse-limit construction, could be made $\mathfrak{c}$ long, at least if MA holds. We see that this is not possible, even if the continuum is separable.

The paper is organized as follows. Section 1 contains a few preliminaries, including the consequences of OCA that we shall use. In Section 2 we construct the continuum K and show how OCA implies that it is not an $\mathbb{H}^*$-image. Finally, in Section 4 we give a few more details on the lack of efficacy of MA in this, and we discuss and ask whether other potential $\mathbb{H}^*$-images are indeed $\mathbb{H}^*$-images.

We thank the referee for pointing out that there was much room for improvement in our presentation.

1. Preliminaries

Closed and open sets in βX. Since we will be working with subsets of the plane we can economize a bit on notation and write βF for the closure-in-βX of a closed subset of the space X itself; we also write $F^* = \beta F \setminus F$. If O is an open subset of X, then $\operatorname{Ex} O = \beta X \setminus \beta(X \setminus O)$ is the largest open subset of βX whose intersection with X is O.

In dealing with closed subsets of $\mathbb{H}^*$ the following, which is Proposition 3.2 from [8], is very useful.

Proposition 1.1. *Let F and G be disjoint closed sets in $\mathbb{H}^*$. There is an increasing and cofinal sequence sequence $\langle a_k : k \in \omega\rangle$ in $\mathbb{H}$ such that $F \subseteq \operatorname{Ex}\bigcup_k (a_{2k+1}, a_{2k+2})$ and $G \subseteq \operatorname{Ex}\bigcup_k (a_{2k}, a_{2k+1})$.* □

We shall be working with closed subsets of the plane (or $\mathbb{H}$) that can be written as the union of a discrete sequence $\langle F_n : n \in \omega\rangle$ of compact sets. The extension of the natural map π from $F = \bigcup_n F_n$ to ω, that sends the points of F_n to n, partitions βF into sets indexed by $\beta\omega$: for $u \in \beta\omega$ we write $F_u = \beta\pi^{\leftarrow}(u)$. If the F_n are all connected, then so is every F_u and, indeed, the F_u are the components of βF; see Corollary 2.2, [8].

For use below we note the following.

Lemma 1.2. *If each F_n is an irreducible continuum, between the points a_n and b_n, say, then so is each F_u, between the points a_u and b_u.* □

The Open Colouring Axiom. The *Open Coloring Axiom* (OCA) was formulated by Todorčević in [11]. It reads as follows: if X is separable and metrizable and if $[X]^2 = K_0 \cup K_1$, where K_0 is open in the product topology of $[X]^2$, then *either* X has an uncountable K_0-homogeneous subset Y *or* X is the union of countably many K_1-homogeneous subsets.

One can deduce the conjunction of OCA and MA from the *Proper Forcing Axiom* or prove it consistent in an ω_2-length countable support proper iterated forcing construction, using $\Diamond$ on ω_2 to predict all possible subsets of the Hilbert cube and all possible open colourings of these, as well as all possible ccc posets of cardinality $\aleph_1$.

We shall make use of OCA only, but we noted the compossibility with MA in order to substantiate the claim that the latter principle does not imply that all separable continua are $\mathbb{H}^*$-images.

Triviality of maps. We shall use two consequences of OCA. The first says that continuous surjections from ω^* onto $\beta\omega$ are 'trivial' on large pieces of ω^*. If $\varphi : \omega^* \to \beta\omega$ is a continuous surjection, then it induces, by Stone duality, an embedding of $\Phi : \mathcal{P}(\omega) \to \mathcal{P}(\omega)/\mathit{fin}$ by $\Phi(A) = \varphi^{\leftarrow}[\beta A]$. The following is a consequence of Theorem 3.1, [6], where for a subset M of ω we write $\tilde{M} = (M-1) \cup M \cup (M+1)$.

Proposition 1.3 (OCA). *With the notation as above there are infinite subsets D and M of ω and a map $\psi : D \to \tilde{M}$ such that for every subset A of $\tilde{M}$ one has $\Phi(A) = \psi^{\leftarrow}[A]^*$.* □

Thus, on the set D^* the map φ is determined by the map $\psi : D \to \tilde{M}$; this is the sense in which $\varphi \restriction D^*$ might be called trivial. It is also important to note that $D =^* \psi^{\leftarrow}[\tilde{M}]$, which follows from $D^* = \varphi^{\leftarrow}[\beta\tilde{M}]$; this will be used in our proof.

Non-images of $\mathbb{N}^*$. The final nail in the coffin of a putative map from $\mathbb{H}^*$ onto the continuum K will be the following result from [5], where $\mathbb{D} = \omega \times (\omega + 1)$.

Proposition 1.4 (OCA). *The Čech-Stone remainder $\mathbb{D}^*$ is not an $\mathbb{N}^*$-image.* □

2. The non-image

The example. We start by replicating the $\sin\frac{1}{x}$-curve along the x-axis in the plane: for $n \in \omega$ we set $K_n = \bigl(\{n\} \times [-1,1]\bigr) \cup \bigl(\{\langle n+t, \sin\frac{\pi}{t}\rangle : 0 < t \leq 1\}\bigr)$. The union $K = \bigcup_n K_n$ is connected, and its Čech-Stone compactification βK is a separable continuum. We shall show that OCA implies that βK is not a continuous image of $\mathbb{H}^*$.

We define four closed sets that play an important part in the proof. For $n \in \omega$ we define:

- $S_n = \{\langle x,y\rangle \in X : n + \frac{1}{3} \leq x \leq n + \frac{2}{3}\}$,
- $S_n^+ = \{\langle x,y\rangle \in X : n + \frac{1}{4} \leq x \leq n + \frac{3}{4}\}$,
- $T_n = \{\langle x,y\rangle \in X : n - \frac{1}{4} \leq x \leq n + \frac{1}{4}\}$,
- $T_n^+ = \{\langle x,y\rangle \in X : n - \frac{1}{3} \leq x \leq n + \frac{1}{3}\}$;

we put $S = \bigcup_{n\in\omega} S_n$, $S^+ = \bigcup_{n\in\omega} S_n^+$, $T = \bigcup_{n\in\omega} T_n$ and $T^+ = \bigcup_{n\in\omega} T_n^+$. Note that $S \cap T = \emptyset$ and hence $\beta S \cap \beta T = \emptyset$ in βX.

We also note that the four sets S_n, S_n^+, T_n and T_n^+ are all connected and that we therefore know exactly what the components of βS, βS^+, βT and βT^+ are. Note that by Lemma 1.2, each continuum T_u^+ (as well as S_u, S_u^+ and T_u) is irreducible, as each T_n^+ is irreducible (between its end points $\langle n - \frac{1}{3}, -1\rangle$ and $\langle n + \frac{1}{3}, 0\rangle$).

Finally we note that S_n meets the sets T_n^+ and T_{n+1}^+ only, that T_n meets S_{n-1}^+ and S_n^+ only, etcetera. This behaviour persists when we move to the continua S_u, T_u^+, S_u^+ and T_u^+, when we define $u+1$ and $u-1$, for $u \in \omega^*$, in the obvious way: $u+1$ is generated by $\{A+1 : A \in u\}$ and $u-1$ is generated by $\{A : A+1 \in u\}$.

Properties of a potential surjection. Assume $h : \mathbb{H}^* \to \beta K$ is a continuous surjection and apply Proposition 1.1 to the closed subsets $h^\leftarrow[\beta S]$ and $h^\leftarrow[\beta T]$ of $\mathbb{H}^*$ to get a sequence $\langle a_k : k \in \omega\rangle$. After composing h with a piecewise linear map we may assume, without loss of generality, that $a_k = k$ for all k. We obtain $h^\leftarrow[\beta S] \cap \beta \bigcup_{k\in\omega} I_{2k+1} = \emptyset$ and $h^\leftarrow[\beta T] \cap \beta \bigcup_{k\in\omega} I_{2k} = \emptyset$, where $I_k = [k, k+1]$. We write 2ω and $2\omega + 1$ for the sets of even and odd natural numbers, respectively.

The map h induces maps from $(2\omega)^*$ and $(2\omega + 1)^*$ onto $\beta\omega$, as follows. If $u \in (2\omega)^*$, then $h[I_u]$ is a connected set that is disjoint from βT; hence it must be contained in a component of βS^+. Likewise, if $v \in (2\omega + 1)^*$, then $h[I_v]$ is contained in a component of βT^+. Thus we get maps $\varphi_0 : (2\omega)^* \to \beta\omega$ and $\varphi_1 : (2\omega + 1)^* \to \beta\omega$ defined by

- $\varphi_0(u) = x$ iff $h[I_u] \subseteq S^+_x$,
- $\varphi_1(v) = y$ iff $h[I_v] \subseteq T^+_y$.

Lemma 2.1. *The maps φ_0 and φ_1 are continuous.*

Proof. For $k \in \omega$ put $r_k = k + \frac{1}{2}$. Observe that, by connectivity, $h(r_u) \in S^+_x$ iff $h[I_u] \subseteq S^+_x$, so that φ_0 can be decomposed as $u \mapsto r_u \mapsto h(r_u) \mapsto \pi_0(h(r_u))$, where $\pi_0 : \beta S^+ \to \beta\omega$ is the natural map.

The argument for φ_1 is similar. □

The maps φ_0 and φ_1 are not unrelated. Let $u \in (2\omega)^*$ and put $x = \varphi_0(u)$. Then $h[I_u] \subseteq S^+_x$, so that $h[I_{u-1}]$ and $h[I_{u+1}]$ both intersect S^+_x. However, S^+_x intersects only the continua T^+_x and T^+_{x+1}, so that $\varphi_1(u-1), \varphi_1(u+1) \in \{x, x+1\}$. By symmetry a similar statement can be made if $y = \varphi_1(v)$; then $\varphi_0(v-1), \varphi_0(v+1) \in \{y-1, y\}$.

Using these relationships we can deduce some extra properties of φ_0 and φ_1.

Lemma 2.2. *If $u \in (2\omega)^*$, then $\varphi_0(u-2)$ and $\varphi_0(u+2)$ are both in $\{x-1, x, x+1\}$, where $x = \varphi_0(x)$.* □

3. An application of OCA

We apply Proposition 1.3 to the embedding Φ_0 of $\mathcal{P}(\omega)$ into $\mathcal{P}(2\omega)/\mathit{fin}$ defined by $\Phi_0[A] = \varphi_0^\leftarrow[\beta A]$. We find infinite sets $D \subseteq 2\omega$ and $M \subseteq \omega$ together with a map $\psi : D \to \tilde{M}$ that induces Φ_0 on its range: for every subset A of $\tilde{M}$ we have $\Phi_0[A] = \psi_0^\leftarrow[A]^*$. As noted above this implies that $\varphi_0 \restriction D^* = \beta\psi_0 \restriction D^*$.

For $m \in \tilde{M}$ and $u \in (2\omega)^*$ we have the equivalence $\varphi_0(u) = m$ iff $\psi_0^\leftarrow[\{m\}] \in u$. Using the properties of φ_0 stated in Lemma 2.2 we deduce the following inclusion-mod-finite:

$$\bigl(\psi_0^\leftarrow\bigl[\{m\}\bigr] - 2\bigr) \cup \psi_0^\leftarrow\bigl[\{m\}\bigr] \cup \bigl(\psi_0^\leftarrow\bigl[\{m\}\bigr] + 2\bigr) \subseteq^* \psi_0^\leftarrow\bigl[\{m-1, m, m+1\}\bigr] \subseteq D.$$

Therefore we get for every $m \in M$ a j_m such that if $n \geq j_m$ and $\psi_0(n) = m$, then $n-2, n+2 \in D$.

Lemma 3.1. *For every $m \in M$ there are infinitely many $n \in D$ such that $\psi_0(n) = m$ and $\psi_0(n+2) \neq m$.*

Proof. Let $m \in M$ and take $m' \in M \setminus \{m\}$. Let $n \in D$ be arbitrary such that $\psi_0(n) = m$ and $n \geq j_m$; choose $n' > n$ such that $\psi_0(n') = m'$. There must be a first index i such that $\psi_0(n+2i) \neq \psi_0(n+2i+2)$, as otherwise we could show inductively that $n + 2i \in D$ and $\psi_0(n+2i) = m$ for all i, which would imply that $\psi_0(n') = m$. For this minimal i we have $\psi_0(n+2i) = m$ and $\psi_0(n+2i+2) \neq m$. □

We use this lemma to find an infinite subset L of D where φ_0 and φ_1 are very well-behaved.

Let $m_0 = \min M$ and choose $l_0 \geq j_{m_0}$ such that $\psi_0(l_0) = m_0$ and $\psi_0(l_0+2) \neq m_0$. Proceed recursively: choose $m_{i+1} \in M$ larger than $m_i + 3$ and $\psi_0(l_i + 2) + 3$, and then pick l_{i+1} larger than l_i and $j_{m_{i+1}}$ such that $\psi_0(l_{i+1}) = m_{i+1}$ and $\psi_0(l_{i+1}+2) \neq m_{i+1}$.

Consider the set $L = \{l_i : i \in \omega\}$ and thin out M so that it will be equal to $\{m_i : i \in \omega\}$. Let $u \in L^*$ and let $x = \varphi_0(u) = \psi_0(u)$; we assume, without loss of generality, that $\{l \in L : \psi_0(l + 2) = \psi_0(l) + 1\}$ belongs to u. It follows that $\varphi_0(u + 2) = x + 1$, and this means that $\varphi_1(u + 1) = x$.

We find that $h[I_u] \subseteq S^+_x$, $h[I_{u+1}] \subseteq T^+_x$ and $h[I_{u+2}] \subseteq S^+_{x+1}$. Therefore the image $h[I_u \cup I_{u+1} \cup I_{u+2}]$ is a subcontinuum of $S^+_x \cup T^+_x \cup S^+_{x+1}$ that meets S^+_x and S^+_{x+1}. Because T^+_x is irreducible we find that $T^+_x \subseteq h[I_u \cup I_{u+1} \cup I_{u+2}]$ and hence $T_x \subseteq h[I_{u+1}]$, because the other two parts of this continuum are disjoint from βT.

We now have infinite sets L and M where the maps φ_0 and φ_1 behave very nicely indeed. Because ψ_0 maps L onto M the map φ_0 maps L^* onto M^*. Furthermore, if $u \in L^*$ and $x = \varphi_0(u)$, then also $x = \varphi_1(u + 1)$ and $T_x \subseteq h[I_{u+1}] \subseteq T^+_x$.

We put $\mathbb{L} = \bigcup\{I_{u+1} : u \in L^*\}$ and we observe that, by the inclusions above,

$$(*) \qquad \bigcup\{T_x : x \in M^*\} \subseteq h[\mathbb{L}] \subseteq \bigcup\{T^+_x : x \in M^*\}.$$

We put $h_L = h \restriction \mathbb{L}$.

A map from $\mathbb{N}^*$ onto $\mathbb{D}^*$. We now use h_L to create a map from $\mathbb{N}^*$ onto $\mathbb{D}^*$, which will yield the contradiction that finishes the proof.

Let $F = \bigcup_{m\in M} T_m \cap \big([0, \infty) \times [\frac{1}{2}, 1]\big)$ and $G = \bigcup_{m\in M} T_m \cap \big([0, \infty) \times [0, 1]\big)$. Observe that the inclusion map from F to G induces the identity map between their respective component spaces and hence also the identity map between the component spaces of βF and βG. We work with the closed subsets F^* and G^* of K^*. The former is contained in the interior of the latter; hence the same holds for $h_L^{\leftarrow}[F^*]$ and $h_L^{\leftarrow}[G^*]$. We apply Proposition 1.1 and obtain, for every $l \in L$, a finite family $\mathcal{I}_l$ of subintervals of I_l such that for the closed set $H = \bigcup_{l\in L} \bigcup \mathcal{I}_l$ we have

$$(\dagger) \qquad h_L^{\leftarrow}[F^*] \subseteq \operatorname{int} H^* \subseteq H^* \subseteq \operatorname{int} h_L^{\leftarrow}[G^*].$$

Endow the countable set of intervals $\mathcal{I} = \bigcup_{l\in L} \mathcal{I}_l$ with the discrete topology and let $p \in \mathcal{I}^*$; the corresponding component of H^* is mapped by h_L into a component of G^*. Thus we obtain a map from $\mathcal{I}^*$ into the component space of G^*. This map is onto: let C_G be a component of G^* and let C_F be the unique component of F^* contained in C_G. Because of (†) and (∗) there is a family of components of H^* that covers C_F; all these components are mapped into C_G.

We obtain a map from $\mathcal{I}^*$ onto the component space of G^*. This map is continuous; this can be shown as for the maps φ_0 and φ_1 using midpoints of the intervals and the quotient map from G^* onto its component space. The component space of G itself is $\mathbb{D}$, so that G^* has $\mathbb{D}^*$ as its component space. Thus the assumption that $\mathbb{H}^*$ maps onto βK leads, assuming OCA, to a continuous surjection from ω^* onto $\mathbb{D}^*$, which, by Proposition 1.4, is impossible.

4. Further remarks

4.1. **Comments on the construction.** The proofs in [5, 6] that certain spaces are *not* $\mathbb{N}^*$-images follow the same two-step pattern: first show that no 'trivial' map exists and then show that OCA implies that if there is a map at all, then there must also be a 'trivial' one. In the context of our example it should be clear that there is no map from $\mathbb{H}$ to the plane that induces a map from $\mathbb{H}^*$ onto βK; it would have been nice to have found a map from $\bigcup_{l\in L} I_{l+1}$ to the plane that would have induced h_L, but we did not see how to construct one.

4.2. MA **is not strong enough.** As mentioned in the introduction the principal result of [7] states that every continuum of weight $\aleph_1$ is an $\mathbb{H}^*$-image. In that paper the authors also prove that under MA every continuum of weight less than $\mathfrak{c}$ is an $\mathbb{H}^*$-image. The starting point of that proof was the result of Van Douwen and Przymusiński [4] that, under MA, every compact Hausdorff space of weight less than $\mathfrak{c}$ is an $\mathbb{N}^*$-image. Given such a continuum X, of weight $\kappa < \mathfrak{c}$, one assumes it is embedded in the Tychonoff cube I^κ and takes a continuous map $f : \beta\mathbb{N} \to I^\kappa$ such that $f[\mathbb{N}^*] = X$. What the proof then establishes, using MA, is that f has an extension $F : \beta\mathbb{H} \to I^\kappa$ such that $F[\mathbb{H}^*] = X$. Thus, in a very real sense, one can simply connect the dots of $\mathbb{N}$ to produce a map from $\mathbb{H}^*$ onto X that extends the given map from $\mathbb{N}^*$ onto X.

Since MA and OCA are compossible our example shows that MA does not imply that all separable continua are $\mathbb{H}^*$-images and, a fortiori, that the two proofs from [7] cannot be amalgamated to show that the answer to Faulkner's question is positive under MA, not even for separable spaces.

4.3. **Other images.** As noted in the introduction there are many parallels between the results on $\mathbb{N}^*$-images and those on $\mathbb{H}^*$-images. The example in this paper shows that there is no complete parallelism. There are some results on $\mathbb{N}^*$-images where no parallel has been found or disproved to exist.

We mentioned Przymusiński's theorem from [10] that every perfectly normal compact space is an $\mathbb{N}^*$-image. By compactness every perfectly normal compact space is first-countable and by Arhangel′skiĭ's theorem ([2]) every first-countable compact space has weight $\mathfrak{c}$ and is therefore an $\mathbb{N}^*$-image if CH is assumed.

Thus we get two questions on $\mathbb{H}^*$-images.

Question 4.1. Is every perfectly normal compact continuum an $\mathbb{H}^*$-image?

Question 4.2. Is every first-countable continuum an $\mathbb{H}^*$-image?

The questions are related of course, but the question on first-countable continua might get a consistent negative answer sooner than the one on perfectly normal continua in light of Bell's consistent example, from [3], of a first-countable compact space that is not an $\mathbb{N}^*$-image.

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