A, the points D, E, and F are the points where that excircle touches BC, CA, and AB, respectively, and the remaining points are defined as in the problem statement.

Solved also by S. Andras (Romania), J. Anglesio (France), R. Barbara (Lebanon), A. M. Barlow, H. J. Barten, M. Benedicty, R. J. Chapman (Australia), J. E. Dawson (Australia), I. Dimitric, D. Donini (Italy), J. Fukuta, W. Janous (Austria), N. Lakshmanan, F. Lang (Switzerland), H.-J. Lee (South Korea), J. H. Lindsey II, O. P. Lossers (The Netherlands), R. Mandl (Austria), R. Merrill, W. W. Meyer, R. D. Nelson (U. K.), A. Nijenhuis, G. Peng, M. Reid, V. Schindler (Germany), A. Schnyder (Switzerland), I. Sofair, T. V. Trif (Romania), M. Woltermann, GCHQ Problems Group (U. K.), and the proposer.

Partitioning the Interval into Closed Sets

10733 [1999, 470]. *Proposed by Sung Soo Kim.* Let $\{E_\alpha\}_{\alpha\in\Omega}$ be a partition of the unit interval $I = [0, 1]$ into nonempty sets that are closed in the usual topology. Is it possible that
(**a**) Ω is uncountable and E_α is uncountable for each $\alpha \in \Omega$?
(**b**) Ω is uncountable but E_α is countably infinite for each $\alpha \in \Omega$?
(**c**) Ω is countably infinite?

Solution by K. P. Hart, University of Technology, Delft, The Netherlands.
(**a**) The answer is yes. Take any continuous square-filling curve $f : I \to I^2$ and let $E_x = f^{-1}(\{x\} \times I)$ (the preimage of the vertical line over x) for $x \in I$. The partition $\{E_x\}_{x\in I}$ is as required.
(**b**) The answer is again yes. We describe a partition of I into sets homeomorphic to the compact set $D = \{0\} \cup \{2^{-n} : n \in \mathbb{N}\}$. Apply Zorn's Lemma to find a maximal disjoint family $\mathcal{C}$ of such sets. The set $A = I \setminus \bigcup \mathcal{C}$ does not contain any sequence of distinct points that converges to a point of A; hence, A is relatively discrete. One can therefore surround the points of A with distinct rational open intervals, and hence A is countable. Now, for each $x \in A$, choose in a one-to-one fashion a set $C_x \in \mathcal{C}$, and replace C_x by $C_x \cup \{x\}$.
(**c**) The answer is no. This follows from Sierpiński's Theorem that no continuum can be partitioned into countably many closed sets (Une théorème sur les continus, *Tôhoku Math. J.* **13** (1918) 300–303).

We provide a direct proof for the space I. Let $\{F_n\}_{n\in\mathbb{N}}$ be a disjoint family of closed nonempty subsets of I. We show that $I \neq \bigcup_{n\in\mathbb{N}} F_n$. For each n, let $\mathcal{O}_n$ denote the family of (nonempty) maximal intervals of $I \setminus \bigcup_{i\leq n} F_i$. We are certainly done if there are an n and an $O \in \mathcal{O}_n$ such that $O \cap F_m = \emptyset$ for every $m > n$, for then $O \cap \bigcup_{i\in\mathbb{N}} F_i = \emptyset$.

In the other case, let O_1 be an element of $\mathcal{O}_1$, and observe that infinitely many F_n must intersect O_1; let F_k and F_l be the first two. Pick $x \in O_1 \cap F_k$ and $y \in O_1 \cap F_l$. We may assume that $x < y$. Now replace x by $x' = \sup\{z \in F_k : z < y\}$, and then replace y by $y' = \inf\{z \in F_l : z > x'\}$. If $k_1 = \max(k, l)$, then the interval $O_{k_1} = (x', y')$ belongs to $\mathcal{O}_{k_1}$ and its endpoints belong to O_1. Continuing in this fashion, we find a sequence $\{k_n\}_{n\in\mathbb{N}}$ of natural numbers, together with intervals $O_{k_n} \in \mathcal{O}_{k_n}$ such that the endpoints of $O_{k_{n+1}}$ belong to O_{k_n}. This ensures that the intersection $\bigcap_{n\in\mathbb{N}} O_{k_n}$ is nonempty. By construction, the intersection is disjoint from $\bigcup_{n\in\mathbb{N}} F_n$.

Editorial comment. Other solvers gave additional references to the literature for part (**c**): exercise 10.2 in R. P. Boas, *A Primer of Real Functions*, MAA, 1960; problem 37 in B. Gelbaum, *Problems in Analysis*, Springer-Verlag, 1982; page 219 in L. A. Steen and J. A. Seebach, *Counterexamples in Topology*, Springer-Verlag, 1978; and G. T. Whyburn, *Analytic Topology*, American Mathematical Society, 1942. A version of the result in part (**a**) appears in G. J. Foschini and R. K. Mueller, On Wiener process sample paths, *Trans. Amer. Math. Soc.* **149** (1970) 89–93.

Solved also by R. Barbara (Lebanon, part **b** only), J. Cobb, G. J. Foschini, O. P. Lossers (The Netherlands), V. Lucic & V. Keselj (Canada), M. D. Meyerson, M. Pulte, K. Schilling, A. W. Schurle, R. B. Tucker, GCHQ Problems Group (U. K.), WMC Problems Group (part **b** only), and the proposer.