Halle the 7^{th} December 73

The last days I have had the time to follow up on the conjecture that I raised with you; today I believe that I am finished with it; should I have deceived myself, however, then I could find no more indulgent judge than yourself. I therefore take the liberty to present for your judgment, what I just committed to paper in the imperfection of a first concept.

One assumes that all positive numbers $\omega < 1$ can be arranged in a sequence

$$\omega_1, \omega_2, \omega_3, \ldots, \omega_n, \ldots \tag{I}$$

Starting with ω_1 let ω_α be the first larger term, after this ω_β is the next larger term, and so on. One puts $\omega_1 = \omega_1^1$, $\omega_\alpha = \omega_1^2$, $\omega_\beta = \omega_1^3$, and so on, and extracts from (I) the following infinite sequence:

$$\omega_1^1, \omega_1^2, \omega_1^3, \ldots, \omega_1^n, \ldots$$

In the remaining sequence one denotes the first term by ω_2^1, the next larger one by ω_2^2, and so on, and thus one extracts a second sequence

$$\omega_2^1, \omega_2^2, \omega_2^3, \ldots, \omega_2^n, \ldots$$

In we continue this then one will realize that the sequence (I) can be decomposed into infinitely many sequences:

$$\omega_1^1, \omega_1^2, \omega_1^3, \ldots, \omega_1^n, \ldots \tag{1}$$

$$\omega_2^1, \omega_2^2, \omega_2^3, \ldots, \omega_2^n, \ldots \tag{2}$$

$$\omega_3^1, \omega_3^2, \omega_3^3, \ldots, \omega_3^n, \ldots \tag{3}$$

in each of these the terms increase continually from left to right, that is,

$$\omega_k^\lambda < \omega_k^{\lambda+1}$$

One now takes an interval (p, q) that contains no terms from the sequence (1); for example inside (ω_1^1, ω_1^2); it is now possible that all terms of the second and even of the third also lie outside (p, q); there must however be a sequence, the k^{th} say, for which not all terms lie outside (p, q); (for otherwise the numbers in (p, q) would not occur in (I), in contradiction with our assumption); then one can fix an interval (p', q') inside (p, q) so that the terms of the k^{th} sequence all lie outside it; of course (p', q') also does not contain any terms of the earlier sequences; there will eventually appear a k'^{th} sequence whose terms are not all outside (p', q') and one will take inside (p', q') a third interval (p'', q'') so that all terms of the k'^{th} sequence lie outside it.

Thus one sees that it is possible to make an infinite sequence of intervals

$$(p, q),\ (p', q'),\ (p'', q''), \ldots$$

in which each contains the next and whose relationship with the sequences (1), (2), (3), ... is as follows:

The terms of the 1^{st}, 2^{nd}, ..., $k - 1^{\text{st}}$ lie outside (p, q)
those of the k^{th}, ..., $k' - 1^{\text{th}}$ lie outside (p', q')
those of the k'^{th}, ..., $k'' - 1^{\text{th}}$ lie outside (p'', q'')

There can now be determined *at least* one number, I call it η that belongs to the interior of all these intervals; of this number, that is clearly > 0 and < 1 one readily sees that it does not occur in any of our sequences (1), (2), ..., (n). Thus one would, assuming that all numbers > 0 and < 1 occur in the sequence (I), be lead to the opposite result that a certain number η that is > 0 and < 1 could *not* be found among the terms of (I); consequently the assumption was erroneous.

Thus I believe I have finally found the reason why the entity that I denoted by (x) in my earlier letters can not be put into correspondence with that which I denoted (n).