AM2520-H: Astronomie - Trigonometrie

week 1.1, dinsdag

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Outline

Newgrange, Ierland

Gedateerd op ongeveer 3200 BC (ouder dan de pyramiden en Stonehenge)

Newgrange, Ierland

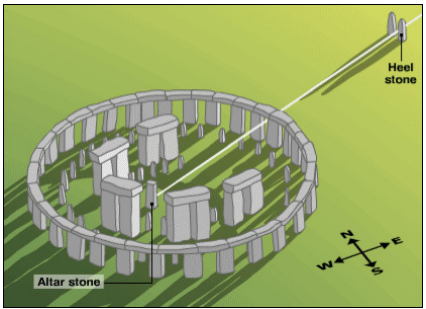
Ingang

Zoninval tijdens Winterzonnewende

Waarom gebouwd?
Veel interpretaties maar er is teruggerekend:
in die tijd scheen de zon bij zonsopgang precies recht de ingang in.
(Tegenwoordig: vier minuten na zonsopgang.)

Stonehenge, Engeland

Bij de Zomerzonnewende



Schematisch

Hier gaat het juist om de zonnewende in de zomer.

Synodisch

De belangrijkste astronomische cycli:

- Maan: cyclus van 29,5306 dagen (= 1 *synodische* maand)
- Zon: cyclus van 365,2422 *synodische* dagen

Synodisch: periode van één cyclus zoals gezien ten opzichte van twee objecten; meestal Aarde en Zon.

De Metonische cyclus

(Meton, ongeveer 400 BC): 235 synodische maanden komt overeen met 19 jaar

$$235 \times 29{,}5306 = 6939{,}691$$
$$19 \times 365{,}2422 = 6939{,}602$$

Toepassing: Het voorspellen van zons- en maansverduisteringen

Egypte

1 jaar $= 365$ dagen $= 12 \times 30 + 5$ extra dagen.

De Seizoenen verschuiven per jaar een kwart dag

Dat geeft een cyclus van $4 \times 365 = 1460$ jaar

Babylonië

Babylonische kalender (ongeveer 8ste eeuw BC)

1 jaar $= 365{,}25$ dagen

Maanden afwisselend 29 en 30 dagen

Cyclus van 19 jaar $= 235$ maanden $= (19 \times 12) + 7$ maanden

Per cyclus 7 schrikkeljaren met 13 maanden

Onze kalender

Juliaans/Gregoriaans (1ste eeuw/1582):

1 jaar $=$ 365,25 dagen (Juliaans)

1 jaar $=$ 365,2425 dagen (Gregoriaans)

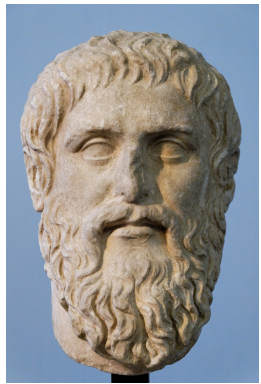
Een model voor de Kosmos

Plato, Eudoxos, Aristoteles

geocentrisch

cirkelvormige banen

uniforme snelheid



Plato (ong. 425–350 BC)

Banen van hemellichamen zijn cirkels

Aristoteles, *On the Heavens* (ca. 350 BC)

"The shape of the heaven is of necessity spherical; for that is the shape most appropriate to its substance and also by nature primary."

"Again, if the motion of the heaven is the measure of all movements whatever in virtue of being alone continuous and regular and eternal, and if, in each kind, the measure is the minimum, and the minimum movement is the swiftest, then, clearly, the movement of the heaven must be the swiftest of all movements. Now of lines which return upon themselves the line which bounds the circle is the shortest; and that movement is the swiftest which follows the shortest line. Therefore, if the heaven moves in a circle and moves more swiftly than anything else, it must necessarily be spherical."

De aarde is rond

Uit de *Almagest* (boek I.4) van Ptolemaeus (ca. 150 AD):

"For we find that the phenomena at eclipses, especially lunar eclipses, which take place at the same time [for all observers], are nevertheless not recorded as ocurring at the same hour (that is, at an equal distance from noon) by all observers. Rather, the hour recorded by the more easterly observers is always later than that recorded by the more westerly. We find that the differences in the hour are proportional to the distances between the places [of observation]. Hence one can reasonably conclude that the earth's surface is spherical (...)"

De aarde draait niet om haar as

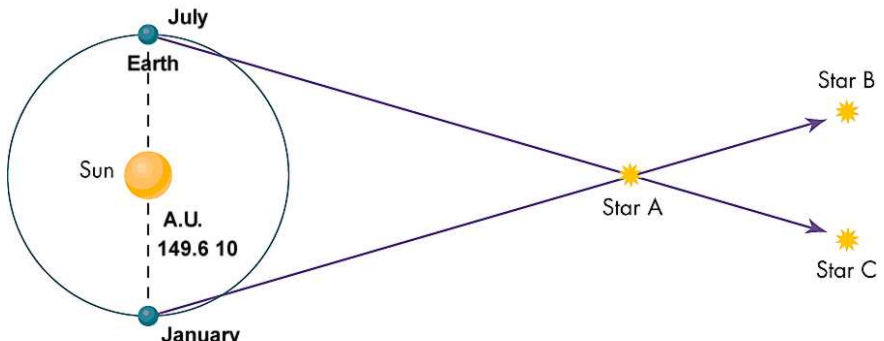
Uit de *Almagest* (boek I.7) van Ptolemaeus:

"(. . .) although there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a motion is quite ridiculous."

"(. . .) they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth: neither clouds not other flying or thrown objects would ever be seen moving towards the east, since the earth's motion towards the west would always outrun and overtake them (. . .)"

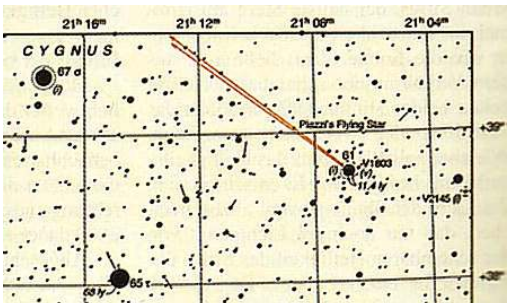
De aarde draait *niet* om de zon

Hipparchos (ca. 250 BC) vond geen (waarneembare) parallax



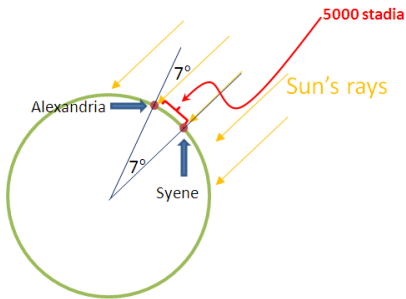
De aarde draait *niet* om de zon

Eerste parallaxmeting door Bessel aan 61 Cygni



Omtrek van de Aarde

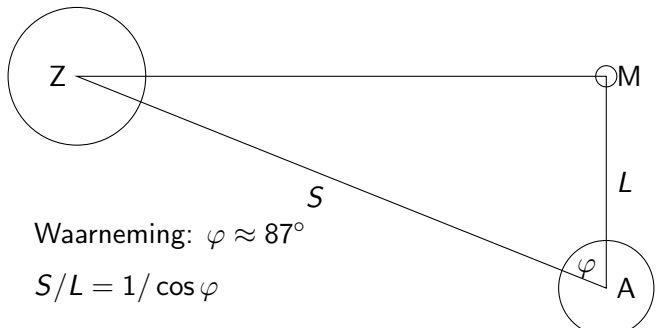
Eratosthenes (276 – ca. 194 BC)



De omtrek van de Aarde is gelijk aan 360/7 maal de afstand Alexandrië–Syene.

Relatieve afstanden tot Maan en Zon

Aristarchus (310 – ca. 230 BC)



Waarneming: $\varphi \approx 87^\circ$

$S/L = 1/\cos\varphi$

Aristarchus bewees: $18 < \frac{1}{\cos 87^\circ} < 20$.

(correcte waarde $S/L \approx 400$)

Schets van het bewijs

Begin met $\cos 87^\circ = \sin 3^\circ$.

Wat men wist was, in moderne notatie: op $(0, \frac{1}{2}\pi)$ geldt

$$\sin x < \tan x$$

en de functies

$$\frac{\sin x}{x} \text{ en } \frac{\tan x}{x}$$

zijn monotoon dalend, respectievelijk stijgend.

Schets van het bewijs

Dus

$$\frac{\sin \frac{1}{60}\pi}{\frac{1}{60}\pi} > \frac{\sin \frac{1}{6}\pi}{\frac{1}{6}\pi} \text{ en dus } \sin \tfrac{1}{60}\pi > \tfrac{1}{60}\pi \cdot \frac{\frac{1}{2}}{\frac{1}{6}\pi} = \frac{1}{20}$$

En ook

$$\frac{\tan \frac{1}{60}\pi}{\frac{1}{60}\pi} < \frac{\tan \frac{1}{8}\pi}{\frac{1}{8}\pi} \text{ en dus } \tan \tfrac{1}{60}\pi < \tfrac{1}{60}\pi \cdot \frac{\frac{1}{1+\sqrt{2}}}{\frac{1}{8}\pi} < \tfrac{1}{60}\pi \cdot \frac{\frac{1}{1+\frac{7}{5}}}{\frac{1}{8}\pi} = \frac{1}{18}$$