AM2520-H: Een beknopte geschiedenis van π

week 1.9, dinsdag

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Outline

Hoe oud is π?

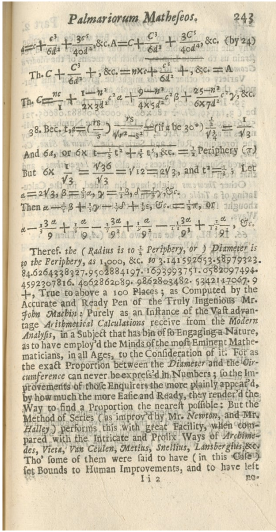
William Jones: *Synopsis Palmariorum Matheseos* (1706) Pagina 263

> There are various other ways of finding the *Lengths*, or *Areas* of particular *Curve Lines*, or *Planes*, which may very much facilitate the Practice; as for Inſtance, in the *Circle*, the Diameter is to Circumference as 1 to $\overline{\frac{16}{5}-\frac{4}{239}} - \frac{1}{3}\,\overline{\frac{16}{5^3}-\frac{4}{239^3}} + \frac{1}{5}\,\overline{\frac{16}{5^5}-\frac{4}{239^5}} -$, &c. $=$ 3.14159, &c. $= \pi$. This *Series* (among others for the ſame purpoſe, and drawn from the ſame Principle) I receiv'd from the Excellent Analyſt, and my much Eſteem'd Friend Mr. *John Machin*; and by means thereof, *Van Ceulen*'s Number, or that in Art. 64. 38. may be Examin'd with all deſireable Eaſe and Diſpatch.

Iets meer dan 300 jaar dus ...

De namen Machin en Van Ceulen komen nog terug.

Nog een pagina



Palmariorum Matheſeos. 243

$a = c + \frac{c^3}{6d^2} + \frac{3c^5}{40d^4}$, &c. $A = C + \frac{C^3}{6d^2} + \frac{3C^5}{40d^4}$, &c. (by 24)

Th. $C + \frac{C^3}{6d^2} +$, &c. $= n \times c + \frac{c^3}{6d^2} +$, &c. $= A$

Th. $C = \frac{nc}{1} + \frac{1-n^2}{2\times 3d^2}c^2\alpha + \frac{9-n^2}{4\times 5d^2}c^2\beta + \frac{25-n^2}{6\times 7d^2}c^2\gamma$, &c.

38. Bec. $t, a = \left(\frac{rs}{s}\right) \frac{rs}{\sqrt{r^2-s^2}} =$ (if a be $30°$) $\frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} = \frac{1}{\sqrt{3}}$

And $6a$, or $6 \times \overline{t - \frac{1}{3}t^3 + \frac{1}{5}t^5}$, &c. $= \frac{1}{2}$ Periphery (π)

But $6 \times \frac{1}{\sqrt{3}} = \frac{\sqrt{36}}{\sqrt{3}} = \sqrt{12} = 2\sqrt{3}$, and $t^2 = \frac{1}{3}$; Let

$\alpha = 2\sqrt{3}$, $\beta = \frac{1}{3}\alpha$, $\gamma = \frac{1}{3}\beta$, $\delta = \frac{1}{3}\gamma$, &c.

Then $\alpha - \frac{1}{3}\beta + \frac{1}{5}\gamma - \frac{1}{7}\delta + \frac{1}{9}\varepsilon$, &c. $= \frac{1}{2}\pi$, or

$\alpha - \frac{1}{3}\frac{3\alpha}{9} + \frac{1}{5}\frac{\alpha}{9} - \frac{1}{7}\frac{3\alpha}{9^2} + \frac{1}{9}\frac{\alpha}{9^2} - \frac{1}{11}\frac{3\alpha}{9^3} + \frac{1}{13}\frac{\alpha}{9^3}$, &c.

Theref. *the* (*Radius is to* $\frac{1}{2}$ *Periphery, or*) *Diameter is to the Periphery, as* 1,000, &c. *to* 3.141592653.58979323.84.6264338327.9502884197.1693993751.0582097494.4592307816.4062862089.9862803482.5342117067.9 +, True to above a 100 Places; as Computed by the Accurate and Ready Pen of the Truly Ingenious Mr. *John Machin*: Purely as an Inſtance of the Vaſt advantage *Arithmetical Calculations* receive from the *Modern Analyſis*, in a Subject that has bin of ſo Engaging a Nature, as to have employ'd the Minds of the moſt Eminent Mathematicians, in all Ages, to the Conſideration of it. For as the exact Proportion between the *Diameter* and the *Circumference* can never be expreſs'd in Numbers; ſo the Improvements of thoſe Enquirers the more plainly appear'd, by how much the more Eaſie and Ready, they render'd the Way to find a Proportion the neareſt poſſible: But the Method of Series (as improv'd by Mr. *Newton*, and Mr. *Halley*) performs this with great Facility, when compared with the Intricate and Prolix Ways of *Archimedes, Vieta, Van Ceulen, Metius, Snellius, Lansbergius*, &c. Tho' ſome of them were ſaid to have (in this Caſe) ſet Bounds to Human Improvements, and to have left

I i 2 no-

Jones has wel een paar opmerkingen:

"The exact proportion between *Diameter* and the *Circumference* can never be exprefs'd in numbers."

"Het werken met reeksen is een stuk makkelijker dan dat gedoe van Archimedes, Vieta, Van Ceulen, Metius, Snellius, . . ."

Daarvoor

Voor Jones werd de verhouding voornamelijk in woorden uitgedrukt of zelfs niet benoemd.

Archimedes, vertaling Heath

The area of a circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius and the other to the circumference of the circle.

Archimedes, vertaling Heath

The ration of the circumference of any circle to its diameter is less than $3\frac{1}{7}$ and larger than $3\frac{10}{71}$.

Bourbaki

3. Dérivées des fonctions circulaires; nombre π

On a défini, en Topologie générale (TG, VIII, p. 8) l'homomorphisme continu $x \mapsto \mathbf{e}(x)$ du groupe additif $\mathbf{R}$ sur le groupe multiplicatif $\mathbf{U}$ des nombres complexes de valeur absolue 1; c'est une fonction périodique de période principale 1, et on a $\mathbf{e}(\frac{1}{4}) = i$. On sait (*loc. cit.*) que tout homomorphisme continu de $\mathbf{R}$ sur $\mathbf{U}$ est de la forme $x \mapsto \mathbf{e}(x/a)$, et qu'on pose $\cos_a x = \mathscr{R}(\mathbf{e}(x/a))$, $\sin_a x = \mathscr{I}(\mathbf{e}(x/a))$ (*fonctions trigonométriques*, ou *fonctions circulaires*, de base a); ces dernières fonctions sont des applications continues de $\mathbf{R}$ dans $[-1, +1]$, admettant a pour période principale. On a $\sin_a(x + a/4) = \cos_a x$, $\cos_a(x + a/4) = -\sin_a x$, et la fonction $\sin_a x$ est croissante dans l'intervalle $[-a/4, a/4]$.

Proposition 3. — *La fonction* $\mathbf{e}(x)$ *admet en tout point de* $\mathbf{R}$ *une dérivée égale à* $2\pi i\mathbf{e}(x)$, *où* π *est une constante* > 0.

En effet, le th. 1 de III, p. 1, appliqué au cas où E est le corps $\mathbf{C}$ des nombres complexes, donne la relation $\mathbf{e}'(x) = \mathbf{e}'(0)\mathbf{e}(x)$; en outre, comme $\mathbf{e}(x)$ a une norme euclidienne constante, $\mathbf{e}'(x)$ est orthogonal à $\mathbf{e}(x)$ (I, p. 15, *Exemple* 3); on a donc $\mathbf{e}'(0) = \alpha i$, avec α réel. Comme $\sin_1 x$ est croissante dans $[-\frac{1}{4}, \frac{1}{4}]$, sa dérivée pour $x = 0$ est $\geqslant 0$, donc $\alpha \geqslant 0$, et comme $\mathbf{e}(x)$ n'est pas constante, $\alpha > 0$; il est d'usage de désigner le nombre α ainsi défini par la notation 2π.

Egypte

Een rond veld met een diameter van 9 khet. Wat is de oppervlakte?

Neem 1/9 weg van de diameter, dat is 1; de rest is 8.
Vermenigvuldig 8 met 8; dat maakt 64. Je hebt dus 64 setat land.

Doe het zo

1	9
$\frac{1}{9}$	1

dit weggenomen laat 8 over

1	8
2	16
4	32
\8	64

De oppervlakte is 64 setat

Mesopotamië

Interpretatie: $O = 3$ (de omtrek dus) en $A = 0;45$ (oppervlakte).

Modern: $2\pi r = 3$ en $\pi r^2 = \frac{45}{60}$; na elimineren van r volgt $\frac{45}{60} = \frac{9}{4\pi}$; dus $\pi = 3$.

Er zijn tabletten met $\pi \approx 3\frac{1}{8}$ (met ingeschreven zeshoek).

De Grieken

Archimedes: $3\frac{10}{71} < \pi < 3\frac{1}{7}$ (door middel van 96-hoeken)

Ptolemaeus: $\pi \approx 3\frac{17}{120}$ (via een 360-hoek)

In hun werk zie betrekkingen tussen lengten van zijden van ingeschreven (a_n) en omgeschreven (b_n) regelmatige n-hoeken.

$$\begin{cases} a_{2n} = \sqrt{a_n b_{2n}} \\ \dfrac{1}{b_{2n}} = \dfrac{1}{2}\left(\dfrac{1}{a_n} + \dfrac{1}{b_n}\right) \end{cases}$$

China

$\pi \approx 3.162$	$(\approx \sqrt{10})$	Zhang Heng	(78–138)
$\pi \approx 3.156$	$(\approx \frac{142}{45})$	Wan Fan	(219–257)
$\pi \approx 3.14159$	(n-hoek)	Liu Hui	(263)
$\pi \approx 3.141593$	$(\approx \frac{355}{113})$	Tsu Chung-Chih	(430–501)
$\pi \approx 3.1415926$	(n-hoek)	Tsu Chung-Chih	(430–501)

India, Islam, Japan

$\pi \approx 3.1416$	(n-hoek)	Aryabhata	($\sim$500)
π (11 dec)	(n-hoek)	Madhava	($\sim$1400)
$\pi \approx 3.1416$	(India?)	Al Khwarizmi	($\sim$800)
π (16 dec)	(n-hoek)	Al Kashi	(1430)
π (10 dec)	(reeks)	Seki	($\sim$1700)
π (41 dec)	(reeks)	Takebe	(1723)

Europa

π (6 dec)	$(\frac{355}{113})$	A. Anthonizoon	(1585)
π (35 dec)	(2^{62}-hoek)	Ludolf van Ceulen	(1610)

Denk terug aan de opmerking van Jones over 'het gedoe'

π (100 dec)	(arctan)	J. Machin	(1706)
π (528 dec)	(arctan)	W. Shanks	(1853)

Producten

F. Viète (∼1600)

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2+\sqrt{2}}}{2} \times \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \times \cdots$$

J. Wallis (1655)

$$\frac{\pi}{2} = \frac{2}{1} \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \cdots$$

Sommen

Madhava (1360–1450) en G. Leibniz (1682)

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

J. Machin (1706) (zie stukjes uit het boek van Jones)

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan 1239$$

L. Euler (1748)

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

Kettingbreuk

W. Brouncker (1655)

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \ddots}}}}$$

Voordrachten

- het werk van Archimedes
- het werk van Van Ceulen
- het Buffon-experiment
- irrationaliteit van π