

I-J function slide rule

User Manual

Shanghai Slide Rule Factory

I-J Function Slide Rule Instructions

With the initiative and support of Hefei University of Technology and Fudan University, our factory has jointly developed the I-J function sliderule for the convenience of use by scientific and technological personnel and college and high school students. Content includes multiplication and division, square, cube, logarithm, trigonometric function, I-J function, hyperbolic function and other scales. Can quickly solve many mathematical problems including cubic and quartic equations. It is very convenient for electrical, mechanics, civil engineering and other calculations, and is a good calculation tool for scientific and technological personnel and college and high school students.

1. Scales

The I-J function slide rule has 26 scales, which adds G, I, I_m , J, J_m , and $1/3x$ scales compared to the general slide rule. It maintains the functions of the 1004 general slide rule and increases the ability to solve third- and fourth-order equations. The first line on the left end of the C ruler is called the left index (C:1), and the first line on the right end is called the right index (C:10). The thin line on the cursor is called the hairline, which is used to indicate numbers during calculations. Scales C and D are also called base scales. The numbers marked on the ruler are significant figures. The readings between the two lines can be obtained by visual interpolation. The position of the decimal point is determined according to the actual situation during the calculation.

2. Multiplication and division

Multiplication and division operations can generally be completed using C and D scales, and both sides of the C and D scales can be used.

Example 1: $2 * 4 = (8)$

Move the slider so that C:1 faces D:2, move the hairline to C:4, and read D:8 below the hairline.

Example 2: $18 / 3 = (6)$ Move the slider so that C: 3 is opposite to D: 18, and D: 6 is read under C: 10.

Example 3:

$(284 * 5.19)/65.2 = (22.6)$, [estimated $(280/70) \times 5 = 20$]

Move the slide ruler so that :65.2 is opposite to D: 284, move the hairline to C:5.19, and read D:22.6 below the hairline.

3. Reciprocal scale red DI

The red DI is the reciprocal of the D scale, and it is read from right to left. When the hairline covers any number X on the D scale, reading the red DI scale under the hairline gives the value of $1/x$. In continuous operations, the red DI scale can be used to convert multiplication into division (or vice versa), thereby simplifying the calculation procedure.

Example 4: $18.5 * 6.2 * 4.75 = 18.5 / (1/6.2) * 4.75 = (545)$ [estimated: $20 * 6 * 5 = 600$]

Set the hairline at DI:6.2, move the sliding ruler so that C:18.5 is also covered by the hairline, move the hairline to D:4.75, and read C:545 under the hairline.

4. Usage of A, B, D and K scales

1. If the hairline covers any number X on the D scale, then the A scale under the hairline will be X^2 and the K scale will be X^3 .

For example: $4^2 = (16)$, $4^3 = (64)$.

2. If the hairline covers any number X of scale A, then scale D below the hairline will be $\text{sq.root}(X)$. For example: $\text{sq.root}(16) = (4)$.

3. If the hairline covers any number X of scale K, then scale D below the hairline will be $\text{cub.root}(X)$. For example: $\text{cub.root}(27) = (3)$.

4. Make C:1 face D:X, and cover the hairline with B:Y, then the A scale under the hairline is X^2Y . For example: $1.5^2 * 4 = 90$

5. If the hairline covers any number X of scale A, then scale K below the hairline is $X^{3/2}$, for example $16^{3/2} = (64)$

5. Trigonometric functions

This slide rule has three trigonometric function scales, namely T, S, and ST. The black numbers on the scale represent the angle of the positive function, and the red numbers represent the angle of the complementary function. Place the hairline at any angle X on the trigonometric function scale, read the C scale below the hairline, and you will get the function value of that angle. During the inverse operation, the angle X can be calculated from the known function value. For example:

$\sin 30^\circ = \cos 60^\circ = (0.5)$; $\text{tg}30^\circ = \text{ctg}60^\circ = (0.577)$; $\sin^{-1}(0.53) = (32^\circ)$.

When $0.573^\circ < X < 5.73^\circ$, use ST scale instead of S and T scale. For example, $\sin 2.5^\circ = (0.0436)$, $\text{tg}2^\circ = (0.0349)$

6. Common logarithmic scale L

L is a commonly used logarithmic scale, which is used to check the mantissa of the logarithm of any known number. The first digit of the logarithm is the number of integer digits in the known number minus one.

Example: $\lg 25 = (1.398)$, $\lg 0.25 = (1.398)$.

Set the hairline at D:25 and read the lower part of the hairline at L:0.398.

7. Usage of double logarithms LL_1, LL_2, LL_3

1. Find the natural logarithm of a number greater than 1:

Cover any of LL_1, LL_2, LL_3 with the hairline, read the D scale below the hairline, and you will get the value of \ln . For example: $\ln(20.1) = (3)$; $\ln(1.6) = 0.47$; $\ln(1.032) = (0.0315)$

2. Find the natural logarithm of a number less than 1:

First, use the DI scaler to read the reciprocal ($1/b$) of the real number b that is less than 1, then place the hairline on any real number ($1/b$) of LL_1, LL_2, LL_3 , and read the D scale under the hairline to get the negative value of $\ln(1/b)$.

For example, $\ln 0.0497 = (-3)$; $\ln 0.67 = (-0.4)$; $\ln 0.9608 = (-0.04)$

3. Find a^x .

Cover LL_3 with the hairline, move the slide ruler so that C:1 is also covered by the hairline, move the hairline to C:x, then the reading of LL under the hairline is a^x .
 For example, $3^4=(81)$.

4. Find $a^{1/x}$.

Cover LL_3 : a with the hairline so that C:x is also covered by the hairline, move the hairline to C:1, then the reading of LL_3 under the hairline is 8, which can be obtained through DI.
 For example, $144^{1/2}=(12)$; $144^{-1/2}=(0.0833)$

5. Find the logarithm of any base: For example, $\lg \text{ base } 9 (729) = (3)$. Set the hairline at $LL_3:9$ so that C:1 is also covered by the hairline. Move the hairline to $LL_2:729$ and read 3 on the C scale.

8. Usage of scales G, I₁, I_m, J₁, J_m, 1/3x

Given $X^3+BX^2+CX+D=0$, from B, C, D in the formula can be obtained:

$$h = - (B/3)$$

$$k = (2/3)*Bh^2+Ch+D$$

$$A = C-3h^2 = \pm r^2$$

Reading from scale A to scale D gives $r = |A|^{1/2}$
 Reading from scale A to scale K gives $r^3 = |A|^{3/2}$
 If 16 of scale A is opposite to 4 of scale D= $|16|^{1/2}$
 if 16 of scale A is opposite to about 64 of scale K= $|16|^{3/2}$

$$G = - (k/r^3) \quad (I_1, J_1 \text{ has the same sign as } G)$$

$$A > 0 \quad \rightarrow I_1, I_m$$

$$A = 0 \quad \rightarrow X_{1,2,3} = h - k^{1/3} (1, -(1/2) \pm i \text{ sq.root}(3/4))$$

$$A < 0 \quad \rightarrow J_1, J_{m-}$$

$$I_{2,3} = - (I_1/2) \pm iI_m$$

$$J_{2,3} = - (J_1/2) \pm iJ_m \quad (G < 0.385. \text{ Cancel } i)$$

$$X_{1,2,3} = h + r * (I_{1,2,3} \text{ or } J_{1,2,3})$$

Example 1 $X^3 - 3X^2 + 19X + 37 = 0$

$$h = - (B/3) = - (-3)/3 = 1; \quad h^2 = 1;$$

$$k = (2/3)Bh^2 + Ch + D = -(2/3)*(-3) + 19 + 37 = 54$$

$$A = C - 3h^2 = 19 - 3 = 16 > 0 \rightarrow I_1, I_m.$$

$$r = |A|^{1/2} = 4$$

$$r^3 = 64$$

$$G = - (k/r^3) = - 54/64 = - 0.843$$

Read out:

$$I_1 = - 0.613.$$

$$I_m = 1.132 \quad \rightarrow I_{2,3} = -(I_1/2) \pm iI_m = 0.307 \pm i1.132$$

$$X_{1,2,3} = h + r * I_{1,2,3} = 1 + 4 * \{-0.613; 0.307 \pm i1.132\} = - 1.452; 2.228 \pm i4.53$$

Example 2: $X^3 + 3X^2 - 13X + 7 = 0$

$$h = - (B/3) = -1; \quad h^2 = 1.$$

$$k = (2/3) Bh^2 + Ch + D = 22$$

$$A = -13 - 3 = -16 < 0 \rightarrow J_1, J_m.$$

$$r = |A|^{1/2} = 4$$

$$r^3 = 64$$

$$G = - k/r^3 = - (22/64) = -0.344$$

Read out

$$J_1 = -1.14.$$

$J_m = 0.154$ (black line represents real number, red line represents imaginary number)

$$J_{2,3} = -(J_1/2) \pm J_m = \{0.724 ; 0.416\}.$$

$$X_{1,2,3} = h + r * J_{1,2,3} = -1 + 4 * \{-1.14; 0.724; 0.416\} = \{-5.56; 0.664; 1.896\}$$

Example 3: $X^3 + 1.71X^2 - 5X + 345670 = 0$

$$h = -(B/3) = -0.57; \quad h^2 = 0.325.$$

$$k = (2/3)Bh^2 + Ch + D = 0.37 + 2.85 + 345,670 = 345,673.22$$

$$A = C - 3h^2 = -5 - (3 * 0.325) = -5.975 < 0 \rightarrow J.$$

$$r = |A|^{1/2} = 2.44$$

$$r^3 = 14.6$$

$$G = -k/r^3 = -23,676 = -(28.72)^3.$$

When $|G| > 3$, the scale cannot read it, and the following formula can be used

$$J_1 = u + (1/3u) \quad (u = |G|^{1/3})$$

$$I_1 = u - (1/3u)$$

$$J_{2,3} = -(J_1/2) \pm \text{sq.root}(1 - (3/4) * J_1^2)$$

$$I_{2,3} = -(I_1/2) \pm i * \text{sq.root}(1 + (3/4) * I_1^2)$$

To find the exit value, the value of $1/3u$ can be read directly from the D scale and the $1/3x$ -scale

$$\text{Since } |G| = 23,679 = (28.72)^3.$$

Therefore $u = 28.72$

$$J_1 = -(28.72 + 1/(3 * 28.72)) = -28.73$$

$$\begin{aligned}
X_{1,2,3} &= h + r^* J_{1,2,3} \\
&= -0.57 + 2.44 * \{-28.73, J_2, J_3\} \\
&= \{-70.78, X_2, X_3 \quad (|G| \geq 0.385, \text{ only one root})\}
\end{aligned}$$

Example 4 Root solution of the quartic equation:

According to $f(x) = x^4 + Bx^3 + Cx^2 + Dx + E = 0$

It can be found that:

$$\begin{aligned}
h &= -(B/4) \\
R &= C - 6h^2 = \pm r^2
\end{aligned}$$

$$\begin{aligned}
\alpha &= (4h^3 + 3Bh^2 + 2Ch + D)/r^3 \\
\beta &= f(h)/r^4
\end{aligned}$$

$$\begin{aligned}
S &= |4\beta + (1/3)|^{1/2} \\
t &= \alpha^2 \mp (2/3) * (4\beta - (1/9)) \\
G &= t/S^3.
\end{aligned}$$

$$\text{If } \beta < -(1/12) \rightarrow M^2 = \mp (2/3) + S * I(G)$$

$$\text{if } \beta = -(1/12) \rightarrow M^2 = \mp (2/3) + t^{1/3}.$$

$$\text{if } \beta > -(1/12) \rightarrow M^2 = \pm (2/3) + S * J(G)$$

$$\begin{aligned}
N &= (1/2) * (M^2 \pm (1 - \alpha/M)) \\
\theta &= \beta/N
\end{aligned}$$

According to α, β , we can find M, N, θ and then

$$(y^2 + My + N)(y^2 - My + \theta) = 0$$

So we get $y_{1,2,3,4}$.

Then the roots of the original equation

$$X_{1,2,3,4} = h + r * y_{1,2,3,4},$$

$$\text{Find } X^4 + 4X^3 + 7.56X^2 + 28X - 47 = 0.$$

$$h = -(B/4) = -1; \quad h^2 = 1.$$

$$R = C - 6h^2 = 7.56 - 6 = 1.56 = (1.252)^2$$

$$r = 1.25$$

$$\alpha = (4h^3 + 3Bh^2 + 2Ch + D)/r^3 = (20.88/1.95) = 10.5.$$

$$\beta = f(h)/r^4 = -70.44/2.44 = -28.7$$

$$S = |4\beta + (1/3)|^{1/2} = |4*(-28.7) + (1/3)|^{1/2} = 10.7.$$

$$t = \alpha^2 - (2/3) * (4\beta - (1/9)) = 10.5^2 - (2/3) * (4*(-28.7) - (1/9)) = 186.5.$$

$$G = (t / S^3) = 186.5/1225 = 0.152.$$

$$\beta = -28.7 < -(1/12)$$

$$M^2 = -(2/3) + S * I(G) = -(2/3) + 10.7 * I(0.152) = -0.67 + 10.7 * 0.149 = 0.93$$

$$M = 0.964.$$

$$N = (1/2) * (M^2 + 1 - (\alpha/M)) = (1/2) * (0.93 + 1 - (10.5/0.964)) = -4.48$$

$$\theta = (\beta/N) = (-28.7 / -4.48) = 6.41$$

The two quadratic equations that determine $y_{1,2,3,4}$ are

$$(y^2 + 0.964y - 4.48)(y^2 - 0.964y + 6.41) = 0$$

$$\text{Then } X_{1,2,3,4} = h + ry_{1,2,3,4} = -1 + 1.25 * y_{1,2,3,4}$$

9. Usage of hyperbolic scales Sh_1 , Sh_2 , Ch and Th

1. Find Sh θ . $Sh .39 = (.4)$ $Sh 2.095 = (4)$

Cover the hairline with $Sh_1:.39$ (or $Sh_2:2.095$) and read $D:.4$ (or $D:4$) under the hairline.

2. Find Th θ . $Th .424=(.4)$, cover the hairline with $Th: .424$, read the hairline under $D: .4$,

3. Find Ch θ . $Ch 1.5=(2.35)$, cover the hairline with $Ch:1.5$, and read $D:2.35$ under the hairline.

10. Usage of S-line, V-line, KW and HP

There is KW engraved on the top of the long hairline on the front slider, and HP engraved on the short hairline on the right. They are used to convert KW (power) and HP (horsepower) to each other.

For example, when the long hair line covers $A:25$, the right short hair line covers $A:34$, that is, $25KW=34HP$. On the front of the slide rule, there is a marker indicated by S on the A and B scales between 78 and 79. Its value $(\pi/4)$ is used to multiply d^2 to find the area of the circle $(\pi/4)*d^2$. On the K scale between 520 and 530 is a marker V and its value is $(\pi/6)$, which is used to calculate the volume of the sphere $(\pi/6)*d^3$.

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June 1975