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Thomas Jech

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For Paula, Pavel, and Susanna

Preface

When I wrote the first edition in the 1970s my goal was to present the state of the art of a century old discipline that had recently undergone a revolutionary transformation. After the book was reprinted in 1997 I started contemplating a revised edition. It has soon become clear to me that in order to describe the present day set theory I would have to write a more or less new book.

As a result this edition differs substantially from the 1978 book. The major difference is that the three major areas (forcing, large cardinals and descriptive set theory) are no longer treated as separate subjects. The progress in past quarter century has blurred the distinction between these areas: forcing has become an indispensable tool of every set theorist, while descriptive set theory has practically evolved into the study of $L(\boldsymbol{R})$ under large cardinal assumptions. Moreover, the theory of inner models has emerged as a major part of the large cardinal theory.

The book has three parts. The first part contains material that every student of set theory should learn and all results contain a detailed proof. In the second part I present the topics and techniques that I believe every set theorist should master; most proofs are included, even if some are sketchy. For the third part I selected various results that in my opinion reflect the state of the art of set theory at the turn of the millennium.

I wish to express my gratitude to the following institutions that made their facilities available to me while I was writing the book: Mathematical Institute of the Czech Academy of Sciences, The Center for Theoretical Study in Prague, CRM in Barcelona, and the Rockefeller Foundation's Bellagio Center. I am also grateful to numerous set theorists who I consulted on various subjects, and particularly to those who made invaluable comments on preliminary versions of the manuscript. My special thanks are to Miroslav Repický who converted the handwritten manuscript to LaTeX. He also compiled the three indexes that the reader will find extremely helpful.

Finally, and above all, I would like to thank my wife for her patience and encouragement during the writing of this book.

Prague, May 2002 *Thomas Jech*

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