Group Interaction #12

MasterMath: Set Theory
2021/22: 1st Semester
K. P. Hart, Steef Hegeman, Benedikt Löwe & Robert Paßmann

Every week, there will be one group interaction of roughly one hour. The group interactions take place remotely via Zoom. A group interaction consists of two students who work together on a work sheet in the presence of one of the two teaching assistants (Steef Hegeman or Robert Paßmann). A group does not have to cover the entire work sheet. If you do not finish the work sheet, feel free to return to it later or in the preparation of the exam.

This group interaction is about names.

(1) Review the definition of a $\mathbb{P}$-name. Is the empty set a $\mathbb{P}$-name? Is $\{\emptyset\}$ a $\mathbb{P}$-name?

(2) Write out the names $\check{0}$, $\check{1}$, and $\check{2}$ in full.

(3) Let $p, q, r \in \mathbb{P}$ and put $\tau = \{\langle\emptyset, p\rangle, \langle\{\langle\emptyset, q\rangle\}, r\rangle\}$. Calculate τ_G for each of the possible eight intersections $G \cap \{p, q, r\}$.

(4) Verify that the union of two names is a name.

(5) Verify that $\sigma_G \cup \tau_G = (\sigma \cup \tau)_G$. Did you use that G is a filter?

(6) Assume $p \perp q$ in $\mathbb{P}$. Exhibit a *proper class* of names σ for which $p \Vdash \sigma = \check{\emptyset}$?

(7) Let τ be a $\mathbb{P}$-name such that $\operatorname{dom} \tau \subseteq \{\check{n} : n \in \omega\}$. Define
$$\sigma = \{\langle\check{n}, p\rangle : (\forall q)(\langle\check{n}, q\rangle \in \tau \to p \perp q)\}$$
Prove that $\sigma_G = \omega \setminus \tau_G$ if G is an M-generic filter.

(8) Given a name τ define
$$\pi = \{\langle\rho, p\rangle : (\exists\langle\sigma, q\rangle \in \tau)(\exists r)(\langle\rho, r\rangle \in \sigma \wedge p \leqslant r \wedge p \leqslant q)\}$$
Show that if G is a filter on $\mathbb{P}$ then $\pi_G = \bigcup \tau_G$.

(9) Let τ and σ be a $\mathbb{P}$-names such that $\operatorname{dom} \tau$ and $\operatorname{dom} \sigma$ are subsets of $\{\check{n} : n \in \omega\}$. Construct a name π such that $\pi_G = \tau_G \cap \sigma_G$ for every filter G on $\mathbb{P}$.