Group Interaction #13

MasterMath: Set Theory
2021/22: 1st Semester
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Every week, there will be one group interaction of roughly one hour. The group interactions take place remotely via Zoom. A group interaction consists of two students who work together on a work sheet in the presence of one of the two teaching assistants (Steef Hegeman or Robert Paßmann). A group does not have to cover the entire work sheet. If you do not finish the work sheet, feel free to return to it later or in the preparation of the exam.

More about names and dense sets.
Let M be a countable model of ZFC and $\mathbb{P}$ a partial order in M.
Let $X \in M$ and let τ be a $\mathbb{P}$-name such that $\mathbf{1} \Vdash \tau \subseteq \check{X}$.

(1) Let $\tau' = \{\langle \check{x}, p\rangle : x \in X, p \in \mathbb{P}, p \Vdash \check{x} \in \tau\}$. Verfy that $\mathrm{val}(\tau, G) = \mathrm{val}(\tau', G)$ for all M-generic filters on $\mathbb{P}$.

(2) Given τ' as in the previous exercise choose for every $x \in X$ a maximal antichain A_x in $\{p : \langle \check{x}, p\rangle \in \tau'\}$ and define $\tau'' = \{\langle \check{x}, p\rangle : x \in X, p \in A_x\}$. Verify that $\mathrm{val}(\tau', G) = \mathrm{val}(\tau'', G)$ for all M-generic filters on $\mathbb{P}$.

(3) Assume σ is a $\mathbb{P}$-name and $p \in \mathbb{P}$ is such that $p \Vdash \sigma \subseteq \check{X}$. Construct a $\mathbb{P}$-name π such that $\mathbf{1} \Vdash \pi \subseteq \check{X}$ and $p \Vdash \pi = \sigma$.

(4) Assume $\mathbb{P}$ is countable in M and κ a cardinal in M. Calculate an upper bound in M for the cardinality of the power set of κ in $M[G]$. *Hint*: think of the calculations of $2^{\aleph_0}$ in class.

(5) Assume $\mathbb{P}$ has cardinality $\aleph_1$ in M and that all antichains in $\mathbb{P}$ are countable in M. Let κ a cardinal in M. Calculate an upper bound in M for the cardinality of the power set of κ in $M[G]$.

(6) Try to generalise the previous exercises. Find an upper bound for the cardinality of $\mathcal{P}(\kappa)$ in $M[G]$ that uses κ, the cardinality of $\mathbb{P}$, and a bound for the cardinalities of the antichains in $\mathbb{P}$.