EXERCISES SET THEORY (01)

2022/23

These exercises are not especially hard. This will let you practise writing down the solution and realise how much you commonly take for granted.

Pretend you are writing a letter to a fellow student wherein you explain the solutions.

Opgave 1. Prove that every set M has a subset that is not an element of M.

Opgave 2. Prove the laws that Zermelo stated for union and intersection:

a. $M \cup N = N \cup M$
b. $M \cup (N \cup R) = (M \cup N) \cup R$
c. $(M \cup N) \cap R = (M \cap R) \cup (N \cap R)$
d. $(M \cap N) \cup R = (M \cup R) \cap (N \cup R)$

The following exercises are a bit harder. They are about Dedekind's work, as described in class. So we have an injective $f : X \to X$ that is not surjective, and $x_0 \in X \setminus f[X]$. The set N is the smallest subset of X that contains x_0 and is closed under f.

Opgave 3. Assume Y is a set and $g : Y \to Y$ is injective and not surjective. Let $y_0 \in Y \setminus f[Y]$ and let M be the smallest subset of Y that contains y_0 and is closed under g.

Prove that there is a bijection $b : N \to M$ that satisfies $b(x_0) = y_0$ and $g \circ b = b \circ f$. (This shows that (N, f, x_0) and (M, g, y_0) are isomorphic.)

Opgave 4. Explain how to define addition in Dedekind's version of the natural numbers.

Date: Monday 12-09-2022 at 20:44:44 (cest).