EXERCISES SET THEORY (02)

2022/23

1. Let A and B be sets. By ${}^{A}B$ we denote the family of functions with domain A and co-domain B. Prove that ${}^{A}B$ is a set.

2. (This exercise is a 'normal' mathematical exercise to practise with the notion of a well-order.) Let A be the family of finite subsets of $\mathbb{N}$ (the set of natural numbers with its order $<$). Define an order $\prec$ on A as follows: first $\emptyset \prec a$ whenever $a \neq \emptyset$ and for nonempty sets a and b we define

$$a \prec b \text{ if } \begin{cases} \max a < \max b \text{ or} \\ \max a = \max b \text{ and } a \setminus \{\max a\} \prec b \setminus \{\max b\} \end{cases}$$

a. Verify that the definition makes sense: for any two finite sets a and b it takes finitely many steps to decide whether $a \prec b$ or not.
b. Alternatively: prove by mathematical induction that $\prec$ is a total order on the power set of $\{1, \ldots, n\}$.
c. Prove that $\prec$ is a well-order.
d. Determine whether $\langle A, \prec \rangle$ is isomorphic with $\langle \mathbb{N}, < \rangle$.

3. (Another 'normal' mathematical exercise.) Prove: if A is a subset of $\mathbb{R}$ (the real line) that is well-ordered by the standard order of $\mathbb{R}$ then A is countable.

Date: donderdag 29-09-2022 at 13:45:10 (cest).