HOMEWORK SET THEORY (01) 2022-09-19

2022/23

Hand in next week by 14:00 on 2022-09-26, either by hand in class, or by uploading to the course page on `elo.mastermath.nl`.

This homework is about Zermelo's infinite set Z_0, the smallest set that contains $\emptyset$ and that is closed under the map $a \mapsto \{a\}$.

Global hint for most of the problems: show that the set in question contains $\emptyset$ and is closed under $a \mapsto \{a\}$.

1. Prove: if $x \in Z_0$ then $x = \emptyset$ or $x = \{y\}$ for some $y \in Z_0$.

Define A_x for $x \in Z_0$ by $A_\emptyset = \emptyset$ and $A_{\{x\}} = A_x \cup \{x\}$.

2. Prove that A_x is indeed defined for all $x \in Z_0$ and that $A_x \subseteq Z_0$ for all x.

3. Prove: for all x in Z_0 we have: if $y \in A_x$ then $y = \emptyset$ or there is $z \in A_x$ such that $y = \{z\}$.

4. Prove: for all x in Z_0 we have: if $y \in A_x$ then $A_y \subseteq A_x$.

5. Prove: for all x and y in Z_0 we have $A_x \subseteq A_y$ or $A_y \subseteq A_x$.

6. Prove: if $x \in Z_0$ and $A \subseteq A_x$ is nonempty then there is a $y \in A$ such that $A_y \subseteq A_z$ for all $z \in A$.

7. Prove: if $A \subseteq Z_0$ is nonempty then there is a $y \in A$ such that $A_y \subseteq A_z$ for all $z \in A$.

8. Prove the existence of $x \times y$ by applying the Power Set Axiom and a suitable instance of the Separation Axiom.

Date: Monday 19-09-2022 at 10:16:21 (cest).