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**Between  $T_1$  and  $T_2$ .**

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The one-point compactification  $X^+$  of a space  $X$  satisfying (KC) compact sets are closed, or (US) sequential limits are unique, is considered. (That  $T_2 \Rightarrow \text{KC} \Rightarrow \text{US} \Rightarrow T_1$ —hence the title—and that no converse holds is well known.) Results: (i)  $X^+$  satisfies US whenever  $X$  satisfies KC; (ii) when  $X$  satisfies KC,  $X^+$  does also if and only if  $X$  is a  $k$ -space; (iii) when  $X$  satisfies US,  $X^+$  does also if and only if each sequence in  $X$  has a relatively compact subsequence. It is asked whether a locally compact space satisfying US need be Hausdorff. {It need not. Simply take a non-isolated point of a compact Hausdorff space to which only an eventually constant sequence can converge, and double it. However, every locally countably compact sequential space satisfying US is Hausdorff (see the reviewer’s paper “Spaces in which sequences suffice, II”, to appear in *Fund. Math.*).}

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