MR0861506 (88a:54021) 54B20 Plewik, Szymon (PL-SILS)

A theorem on spaces of finite subsets.

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The "Ochan topology" on the space of subsets of a given space X is generated by the sets $[x, V] = \{y \subset X : x \subset y \subset V\}$, where x is a closed subset of X and V is an open subset of X. The collection of nonempty finite subsets of the reals endowed with the Ochan topology was shown to be a Moore space by Pixley and Roy. The author shows that if $\mathcal{F}[x]$ is the collection of nonempty, finite subsets of a T_1 -space X endowed with the induced Ochan topology then the sets $\langle x, V \rangle = [x, V] \cap \mathcal{F}[X]$ are closed-open and form a base. Moreover, it is shown that if λ is a regular cardinal and X is a T_1 -space with no or infinitely many isolated points such that for each point $x \in X$ there exists a decreasing and well-ordered base $\{x(\alpha) : \alpha < \lambda\}$ of open neighborhoods, then the iterated hyperspaces $\mathcal{F}[\mathcal{F}[x]]$ and $\mathcal{F}[\mathcal{F}[\mathcal{F}[x]]]$ of finite subsets endowed with the Ochan topology are homeomorphic.