Analysis in Banach spaces - Volume I

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Errata and corrigenda

page xix line 2: delete "of".

page 5, line -1: the minimum should be taken over $j \in F_k$, where $F_k = \{1 \leq j \leq k : \|x_j\| \leq \|y\|\}$ and a similar correction should be made in the third displayed formula on the next page. Alternatively one could give up worrying about the norm domination, but then Corollary 1.1.7 would need a separate proof.

page 6, Corollary 1.1.7: in the displayed formula, replace all 'x' and '$x \in X$' by 's' and '$s \in S$'.

page 7, It was kindly pointed out to us by Ryan Kurniawan that the proof of Lemma 1.1.12 contains a gap. The proof depends on the assumption that if an index set I is uncountable, then the cardinality of the set 2^I is strictly greater than that of $2^{|\mathbb{N}|}$. In ZFC this is true if one assumes the Continuum Hypothesis, but false if one assumes both Martin's Axiom and the negation of the Continuum Hypothesis (the latter was kindly pointed out by Klaas Pieter Hart).
Lemma 1.1.12 is used to prove Lemma 1.1.11, which in turn is used to prove Corollary 1.1.24. Without additional set theoretic assumptions, Lemma 1.1.11 (with a simpler proof) holds for continuous functions f. Hence, Lemma 1.1.11 and Corollary 1.1.24 hold for continous functions ϕ; both results are used exclusively under this more restrictive assumption and therefore the gap in the proof of Lemma 1.1.11 has no consequences for the rest of the volume.

page 10, Corollary 1.1.21: in the displayed formula, replace all 'x' and '$x \in X$' by 's' and '$s \in S$'.

page 11, line -13: replace "nulls" by "null".

page 15, line -5: replace "proof of the proposition" by "proof of the theorem".

page 18, in Remark 1.2.8, the text "but it cannot be approximated by simple functions having the properties stated above (such an approximation would involve at most countably many points of T)" should be deleted.

page 21, line -10: replace "$L^\infty(S;X)$" by "$L^p(S;X)$"; line -8: replace "f_{k_k}" by "f_{n_k}".

page 22, in the formulation of Lemma 1.2.18, 'L^1' should read 'L^p' twice.

page 28, in Lemma 1.2.30, we can take $1 \leq p \leq \infty$, but a separate argument should be added for the case $p = \infty$.

page 33, last line of the proof: replace "f" by "ϕf".

page 38, line 10: after "of radius ε" add "of strictly positive μ-measure"; line -10: replace "form" by "for".

page 39, lines 6 and 7: replace "$\ell^r(X)$" by "$\ell^r(I;X)$".

page 45, line 7: "theorem" by "Theorem".

page 76, lines -7 and -8: replace "as a bounded operator the spaces $L^p(X_{\mathbb{C}})$, has the same norm as $T \otimes I_X$" by "as a bounded operator on the spaces $L^p(X_{\mathbb{C}})$, has the same norm as $T \otimes I_X$ on $L^p(X)$".

page 80, line 3: replace "$e^{-2\pi ikx}$" by "$e^{-2\pi inx}$".

page 81, paragraph starting with "Analogous": replace "discrete" by "periodic" (twice).

page 84, in the formulation of lemma 2.2.2: A mild *a priori* growth condition on F is missing which is needed to ensure the asserted decay of $F_\varepsilon(u+iv)$. Indeed it is well known that the three lines lemma is wrong without such a condition: the function $\exp(\exp(\pi i(z - \frac{1}{2})))$ is bounded on the lines $\Re z = 0$ and $\Re z = 1$, but unbounded on the line $\Re z = \frac{1}{2}$. In the applications of the lemma, the functions under consideration are always bounded; line -7: replace "A_0^θ" by "A_1^θ" twice.

page 86, line -9: replace "$Tf)$" by "$T(f)$".

page 90, line -11: delete the word "optimal".

page 99, line 5: replace "ad" by "and".

page 104, line 9: replace "dy"' by "dz".

page 108, line 3: it should be added that $\breve{L}^1(\mathbb{R}^d; X)$ is a Banach space with respect to the norm $\|\breve{f}\|_{\breve{L}^1(\mathbb{R}^d;X)} := \|f\|_{L^1(\mathbb{R}^d;X)}$; Lemma 2.4.7: replace "$\mathbb{R}$" by "$\mathbb{R}^d$" (5 times in the statement, once in the proof).

page 110, line 7: replace "operator on norm at most one" by "operator of norm at most one".

page 113, Proposition 2.4.17: in the second displayed formula in the statement of the result, "$\varphi_{(X_0,X_1)_{\theta,p_0,p_1}}$" should be replaced by "$\varphi_{(X_0,X_1)_{\theta,p_0,p_1}}(\mathbb{R}^d)$"; in the statement and its proof, some of the "p" should read "p_θ".

page 115, part (3) of the proposition: replace the second "p" by "p'"; in the first displayed formula of the proof, replace "$\mathbb{T}^d$" by "$\mathbb{Z}^d$".

page 123, line 14: replace "$f \in L^1_{\text{loc}}(\mathbb{R}^d; X)$" by "$f \in L^1_{\text{loc}}(D; X)$"; line 15: after "almost everywhere" add "on D"; proof of Proposition 2.5.3: one has to change the proof a bit, since in Step 2 ψf does not have a vanishing gradient outside B.

page 124, line 7: replace "D^α" by "∂^α"; in the formulation of Lemma 2.5.5 the assumption "let $f \in L^p(\mathbb{R}^d; X)$ be such that Af exists in the weak sense and belongs to $L^p(\mathbb{R}^d; X)$; in line 2 of the proof replace "$L^p(\mathbb{R}^d)$" by "$L^p(\mathbb{R}^d; X)$".

page 125: we learnt that part (2) of Proposition 2.5.7 has been obtained independently, with a different proof, by Marcel Kreuter as part of his 2015 M.Sc. thesis submitted to the University of Ulm. This proof will appear in a forthcoming paper by Wolfgang Arendt and Marcel Kreuter in Studia Math. (arXiv:1611.06161). This paper contains a wealth of interesting material on vector-valued Sobolev spaces not covered in our monograph.

page 131: line -3: replace "$0 = t_0 < \cdots < t_N = 1$" by "$0 = t_0 < \cdots < t_N = t$".

page 134, line 3: replace "$W^{s,p}(\mathbb{R}^d; X)$" by "$W^{s,p}(D; X)$"; line 7: there should be a double integral.

page 152, line 7: replace "Turing" by "Turning".

page 155, line 14: replace "$S|V|$" by "$V|x|$"; Theorem 2.7.3: It is implicit here that the scalar field is $\mathbb{R}$.

page 158, 5 lines before Theorem 2.7.5: replace "such" by "Such".

page 158, line 3 of Theorem 2.7.5: replace "$p_0 < p_0$" by "$p_0 < p_1$".

page 162, line 9: replace "gives" by "given".

page 164, line 12: replace "$L^\infty(S; L^2(T))$" by "$L^p(S; L^2(T))$". The correct norm in the same line reads $(\frac{1}{2}(\frac{1}{8})^{p/2} + \frac{1}{2}(\frac{5}{8})^{p/2})^{1/p}$.

page 166, line -16: replace "Such a martingales is called if" by "Such a martingale is called L^p-*bounded* if".

page 171, Gilles Pisier has kindly pointed out that the result quoted as Theorem 2.7.7 is wrong as stated. The arXiv preprint from which it was taken has been replaced by a corrected version.

page 175, Theorem 3.2.2: in the first assertion of (3.6) one should add the condition $\lambda > 0$.

page 182, in (3.17), replace "$L^p(\Omega; X)$" by "X" twice; line -5: replace "is obtained as in (3.22)" by "can be obtained as explained below in (3.22)".

page 194, proof of Lemma 3.2.26: In the second paragraph, "$E_k^\alpha = \varnothing$" should be replaced by "$E_k^\alpha \cap I = \varnothing$". In the third paragraph, "$\ell(I_i) < \ell(J_i) \leq 6\ell(I_i)$" should be replaced by "$\frac{3}{2}\ell(I_i) < \ell(J_i) \leq 3\ell(I_i)$".

page 214, Theorem 3.4.1 (Gundy decomposition): replace the equation

$$g_{-\infty} = f_{-\infty}, \qquad b_{-\infty} = h_{-\infty} = 0$$

by

$$g_{-\infty} = f_{-\infty}\mathbf{1}_{\{\|f_{-\infty}\| < \lambda\}}, \quad b_{-\infty} = f_{-\infty}\mathbf{1}_{\{\|f_{-\infty}\| \geq \lambda\}}, \quad h_{-\infty} = 0.$$

(The proof is unchanged; one can see that the given proof actually proves the latter statement, not the first.)

page 230, line -2: on the right-hand side, replace "f" by "fx".

page 260, paragraph starting with "In $\mathbb{R}^d$": replace "$k = 1, \ldots, N$" by "$k = 1, \ldots, N_d$" and "$\ell(Q) \leq C_d \ell(D)$" by "$\ell(D) \leq C_d \ell(Q)$".

page 305, display in the middle of the page: replace "θ" by "ϑ" (twice).

page 284, line -10: replace "$(f_n)_{n=0}^N$" by "$(f_j)_{j=0}^n$".

page 293, in part (5) and its proof some of the "p" should read "p_θ".

page 307, line -4 of the proof of Theorem 4.3.3: replace "ℓ_N^∞ into $L^q(\Omega \times [0,1); X)$ that satisfies $\theta \leq \|J\| \leq \beta_{p,X}^+$" by "$\ell_N^\infty$ into $L^p(\Omega \times [0,1); X)$ that satisfies $\|J\|\|J^{-1}\| \leq \frac{1}{\theta}\beta_{p,X}^+$" and "$\beta_{p,\ell_N^\infty}^+ \leq \frac{1}{\theta}\beta_{p,X}^+$" by "$\beta_{p,\ell_N^\infty}^+ \leq \|J\|\|J^{-1}\|\beta_{p,L^p(X)}^+ \leq \frac{1}{\theta}(\beta_{p,X}^+)^2$".

page 345, line -10: replace the second "dg_n" by "df_n".

page 345, line 7: replace "$\leq$" by "$\geq$".

page 376, line -5: replace "$z \in \Sigma_\eta$" by "$\zeta \in \Sigma_\eta$".

page 400, lines -4 and -3: the subscript "so" should be left out twice.

page 400, line -12: replace "$\mathbb{R}^d$" by "$\mathbb{R}$".

page 403, line -12: replace "X^*" by "Y^*" in the expression on the left-hand side; line -10: remove the '.' at the end of the equation; line -7: the last three 'm' in the equation should be replaced by 'm^*'.

page 406, line 4: two closing brackets are missing.

page 407, line 10: replace "last admissible" by "least admissible".

page 408, proof of Theorem 5.3.15: the sentence "Since m is assumed only locally integrable first ..." can be deleted. This is because we assume that m is uniformly bounded in the definition of a multiplier. If one would only assume local integrability of m, then an additional argument is required: first consider compactly supported $\widehat{\phi}$ and $\check{\psi}$, and after the conclusion that m is uniformly bounded, one can consider arbitrary Schwartz functions.

page 410, lines -11, -10, and -9: replace "$\mathbb{R}^n$" by "$\mathbb{R}^d$".

page 433, Lemma 5.5.5: this lemma holds for arbitrary Banach spaces X.

page 446, line -1: replace "ψ_t" by "$\psi_t(x)$".

page 448, line 12: replace "morms" by "norms".

page 455, line -8: replace "$C_c^\infty(\mathbb{R})$" by "$C^\infty(\mathbb{R})$".

page 469, line 2: replace "$\mathbb{T}$" by "$\mathbb{T}^d$" (three times).

page 493, lines -4, and -2: replace "$\mathbb{R}^n$" by "$\mathbb{R}^d$", replace "$n/2$" by "$d/2$".

page 494, lines 6: replace "$\mathbb{R}^n$" by "$\mathbb{R}^d$".

page 510, line -5: replace "conversely" by "converse".

page 521, line 4: replace "$V(x_0, y_0^*, \varepsilon)$" by "$V(x_0, x_0^*, \varepsilon)$".

page 521, line 10: replace "$\lim_{n\to\infty}\langle x, x_n^*\rangle = \langle x, x_n^*\rangle$" by "$\lim_{n\to\infty}\langle x, x_n^*\rangle = \langle x, x^*\rangle$".

page 526, line –2 of the proof of Proposition B.1.17: replace "$\Re\langle x^*, x^*\rangle$" by "$\Re\langle x, x^*\rangle$".

page 538, line -8: replace "y" by "by".

page 539, Definition C.2.1: the same remark as on page 84 applies. It suffices to add the condition that functions in $\mathscr{H}(X_0, X_1)$ belong to $C_b(\overline{\mathbb{S}}; X_0 + X_1)$.

page 540, line 1: replace "$H(X_0, X_1)$" by "$[X_0, X_1]_\theta$"; line 3: replace "$g_n \in [X_0, X_1]_\theta$" by "$g_n \in H(X_0, X_1)$"; line 3 of the proof of C.2.5: replace "x_1" by "x".

page 546, line -7: replace '$\le$' by '$\lesssim_{\theta, p_0}$'.

page 565, Appendix D: it is implicit that in this appendix, inner products are linear in the second variable and conjugate-linear in the first. This should be reversed to be consistent with the conventions made elsewhere in these volumes.

page 593, line -15: the correct title of this article is "Banach Space-valued Extensions of Linear Operators on L^∞".

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