Analysis in Banach spaces - Volume II

Tuomas Hytönen, Jan van Neerven, Mark Veraar, Lutz Weis

(November 28, 2024)

Errata and corrigenda

page 1, line 11: Delete "the mentioned".
page 2, line -9: After "measurable" add "function".
page 21, line 10: Replace "if" by "only if"; in line 13 replace "Lemma 6.2.3" by "the inequality stated in the formulation of Lemma 6.2.3".
page 39, line 4 of the proof of Theorem 6.4.5: replace "$\overline{B}$" by "$\overline{B}_X$" (as used in the proof) and "cover x" by "cover X".
page 50, Theorem 6.6.3: Delete "X be a Banach space and" from the statement of the theorem.
page 54, line 13: Replace "form" by "from".
page 56, line 8: Replace the first "in" by "is"; throughout the proof: the probability space Ω is missing from the L^p-notation.
page 57, line -5: Replace $L^{r_1}(S_1(L^{r_2}(S_2,\dots$ by $L^{r_1}(S_1;L^{r_2}(S_2;\dots$ (with semicolons instead of a bracket and a comma).
page 58, line -8: Replace "Fubini' " by "Fubini's".
page 102, in the displayed formula on the middle of the page replace "$c_{2,X}$" by "$\tau_{2,X}$" twice.
page 107, line 11: replace "$c \in \overline{(B_{\mathbb{K}})}^N$" by "$c \in (\bar{B}_{\mathbb{K}})^N$".
page 108, Lemma 2.4.7: various occurrences of $\mathbb{R}$ should read $\mathbb{R}^d$; line -1: replace "$\tau_{2/\eta,X}$" by "$c_{2/\eta,X}$".
page 129, line 3: Replace "range" by "image"; line 5: Replace "S" by "T".
page 130, Definition 7.5.1: In the formula, "$\|a\|_\infty$" should be replaced by "$\|a\|_\infty^2$".
page 193, line 3: replace the subscript "b" by "b_J"; line -4: replace "$(t_k)_{k=1}^{2K}$" by "$(t_k)_{k=0}^{2K}$".
page 194, line -7: replace "$1,\dots,K$" by "$1,\dots,K-1$"; line -5: replace "for $j \geq 1$" by "for $j = 1,\dots,K$".
page 201, line -8: replace "$V^1(\mathbb{R};\mathscr{T})$" by "$V^1(R;\mathscr{T})$".
page 202, line -6: replace "$T_{m\mathbf{1}_R}$" and "$T_{m^{(k)}\mathbf{1}_R}$"by "$T_m\mathbf{1}_R$" and "$T_{m^{(k)}}\mathbf{1}_R$".
page 214, line 4: after "This provides a way" add "to".

page 222, formulation of Proposition 8.5.8: in addition to the conditions stated, f should be continuous on $\overline{\Sigma_\sigma}$.
page 224, formulation of Proposition 8.5.10: replace "be simply" by "be a simply"; add the assumption that f be bounded.
page 233, line 2 of the formulation of Theorem 8.5.21: replace "$\frac{1}{r}$" by "$\frac{d}{r}$" .
page 234, line 2: replace the exponent "$s + d/p$ by "$s + d/r$".
page 260, line -9: replace "$L^p(\Omega; Y)$" by "$L^p(\Omega; X)$".
page 265, first line of the proof of Theorem 9.1.20: add "consider" behind "let us first".'
page 279, Proposition 9.2.9(1): "the" should be replaced by "then".
page 284, formula (9.20): On the RHS a term "$\|\gamma\|_{L^p(\Omega)}$" is missing.
page 285, line 1: Replace "spaces" by "space"; Proposition 9.3.2: "$\phi : L^p(S; H^*)''$ should be replaced by "$\phi \in L^p(S; H^*)$".
page 288, line 10: replace "Arguing in" by "Arguing as in".
page 290, in the statement of Lemma 9.3.7 replace "over $(T, \mathscr{B}, \nu)$" by "over $(S, \mathscr{A}, \mu)$".
page 300, Theorem 9.5.1: The range of p is not specified: the theorem holds for all $1 \le p < \infty$.
page 305, Proposition 9.5.6: Replace '$\gamma^p(\mathscr{M})$' by '$\gamma(\mathscr{M})$' in the formulation of the result.
page 306, line -2: delete "We refer to the Notes for a discussion of this point.".
page 322, line -9: replace "$\gamma(I; X)$" by "$\gamma_p(I; X)$".
page 323, line 1: replace "$\tau_{p,X}$" by "$\tau^\gamma_{p,X}$".
page 324, line -9: Replace "a, b" by "I" twice.
page 330, Theorem 9.7.7: "$0 \le a < b < c\alpha$" should be replaced by "$0 \le a < b < c < \alpha$".
page 341, before Proposition 9.7.19: Replace "Propositions 10.4.15 and 10.4.15" by "Propositions 10.4.15".
page 346, line -4: Replace "If" by "It".
page 359, line -4: replace "Banach" by "Banach space".
page 363, line 1: replace "for some $0 < \theta < \frac{1}{2}\pi$" by "for some $0 < \eta < \frac{1}{2}\pi$"; line -8: replace "$H^{2,p}(\mathbb{R}; X)$" by "$H^{2,p}(\mathbb{R}^d; X)$". In this example, it should be assumed that X is a UMD space (see also Example 15.3.3).
page 366, the proof of (3) should refer to Theorem 10.1.7(3) instead of 10.1.7(2).
page 376, line 13-14: It should be added that $\omega(A) < \pi/2$.
page 378, Lemma 10.2.8: It should be added that $\omega(A) < \pi/2$.
page 379, second line after (10.14): replace "$(1+A)^{-1}$" by "$(I+A)^{-1}$"; third line after (10.14): replace "$(1 + A)^{-1}y - y$" by "$(I + A)^{-1}y = y$".
page 384, Lemma 10.2.16(3),(3)′: Replace "$\Sigma_{\sigma-\vartheta}$" by "$\Sigma_{\vartheta-\sigma}$".
page 386, statement of the Proposition 10.2.18: The sentence "Then the part A_Y of A to Y has a bounded H^∞-calculus on X, then A_Y has a bounded H^∞-calculus on Y with ..." should be replaced by "If A has a bounded H^∞-calculus on X, then A_Y has a bounded H^∞-calculus on Y with ..."; line -3: Replace "boundedness of $f(A)$" by "boundedness of $\Psi(f)$ on $L^p(\mathbb{R}^d; X)$".
page 388, in the formulation of Proposition 10.2.22 replace "M_m" by "A_m" twice.
page 391, line 5ff: '$\frac{1}{2\pi i}$' is missing in front of the integrals.
page 392, line 1: Replace "D^α" by "∂^α"; replace "$|t|^{2|\alpha|}$" by "$|t|^{|\alpha|}$".
page 398, line 19: Replace "$H^\infty(\Sigma_{\pi-\delta}$" by "$H^\infty(\Sigma_{\pi-\delta})$".
page 405, proof of Lemma 10.3.8: "applied to the functions $t \mapsto f(tz)$" should be replaced by "applied to the functions $\eta \mapsto f(\eta z)$". This adjustment is required since t is fixed as a real parameter at the beginning of the proof.

page 409, line 4: the constant $C^2_{\theta+\omega}$ is missing behind the + on the right-hand side.

page 415, in the second formula: "$\|f(A)x\| \leq C\|fh\|_{H^\infty(\Sigma_\sigma)}\|x\|$" should be replaced by "$\|f(A)x\| \leq C\|f\|_{H^\infty(\Sigma_\sigma)}\|x\|$".

page 416, first line: Remove "Taking $\epsilon = \varepsilon(\omega)$ and averaging with respect to $\omega \in \Omega$ we obtain (1)"; middle displayed formula: In the second line "$\|x^*\|_{\psi,A}$" should read "$\|x^*\|_{\psi,A^*}$", and in the third line "$\|x\|_{\phi,A^*}$" should read "$\|x\|_{\phi,A}$".

page 417, line 11; "$\{zR(z,A) : |\arg(z)| < \sigma\}$" should be replaced by "$\{zR(z,A) : |\arg(z)| > \sigma\}$."

page 421, last sentence of Proposition 10.4.8: "This proves the upper bound in (10.35). The lower bound (with constant) 1 holds trivially", should be moved to the end of the proof and "(10.35)" should be replaced by the formula stating the equivalence of the norms in the proposition.

page 422, a closing bracket is missing in line 4 of the proof of Corollary 10.4.10.

page 424, the second formula of the page: "$\|t \mapsto \phi(tz)\|_{L^2(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)}$" should be replaced by "$\|t \mapsto \phi(tz)\|_{L^2(\mathbb{R}_+,\frac{\mathrm{d}t}{t})}$".

page 425, statement of Proposition 10.4.15: "$0 < \delta < \omega(A) - \sigma$" should be replaced by "$0 < \delta < \sigma - \omega(A)$".

page 426, Theorem 10.4.16: The hypothesis "X has finite cotype" should be dropped in the main statement.

page 427, before Proposition 10.4.17: "$\{zR(z,A) : z \in \mathbf{C}\overline{\Sigma_\nu}\}$" should be replaced by "$\{zR(z,A) : z \in \mathbf{C}\overline{\Sigma_\sigma}\}$"; in Proposition 10.4.17(2): the formula

$$\|\phi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)} \simeq \|\psi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)}$$

should be replaced by

$$\|t \mapsto \phi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)} \simeq \|t \mapsto \psi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)}.$$

page 429, proof of Theorem 10.4.19: In the chain of inequalities, the second line

$$\leq c_0 C_{\phi,\psi}(M^\gamma_{\sigma,A})^2\|f\|_{H^\infty(\Sigma_\sigma)}\|f\|_\infty\|t \mapsto \phi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)},$$

should be replaced by

$$\leq c_0 C_{\phi,\psi}(M^\gamma_{\sigma,A})^2\|f\|_{H^\infty(\Sigma_\sigma)}\|t \mapsto \psi(tA)x\|_{\gamma(\mathbb{R}_+,\frac{\mathrm{d}t}{t};X)}.$$

Proposition 10.4.20: "$\omega(A) < \sigma < \vartheta < \pi$. For all $\phi \in H^1(\Sigma_\vartheta)$" should be replaced by "$\omega_R(A) < \sigma < \pi$. For all $\phi \in H^1(\Sigma_\sigma)$", and in the proof one should take $0 < \delta < \sigma - \omega_R(A)$ and $\omega_R(A) < \nu < \sigma - \delta$.

page 430, line 10: Omit "of angle less than $\frac{1}{2}\pi$".

page 431, formula in (2) of Theorem 10.4.23: In the integral in the middle term "$\mathrm{d}t$ should be replaced by "$\mathrm{d}t/t$".

page 432, last formula on the right-hand side: Replace "$\|F\|_{H^\infty(\Sigma_\sigma;H^*)}$" by "$\|F\|_{H^\infty(\Sigma_\vartheta;H^*)}$".

page 435, line 6: Replace "$\in\in$" by "$\in$".

page 436, Lemma 10.4.29: In the first formula, the middle term "$\|t \mapsto \Psi(t)x\|_{\gamma(T;X)}$" should be replaced by "$\|t \mapsto \Psi(t)x\|_{\gamma(T;Y)}$".

page 447, line 2 after the figure: Replace "$\mathbb{R}_+$" by "$\mathbb{R}$".

page 449, fourth line after Section 10.6.b: "$f : \Sigma_\sigma^{\rm bi} \to \mathbb{C}$" should be replaced by "$f : \Sigma_\sigma^{\rm bi} \to \mathbb{C}$"; in the second formula:

$$\|f\|_{H^p(\Sigma_\sigma^{\rm bi})} := \sup_{|\nu|<\sigma} \|t \mapsto f(e^{i\nu}t)\|_{L^p(\mathbb{R}_+,\frac{{\rm d}t}{t})}$$

should be replaced by

$$\|f\|_{H^p(\Sigma_\sigma^{\rm bi})} := \sup_{|\nu|<\sigma\,,\,|\nu-\pi|<\sigma} \|t \mapsto f(e^{i\nu}t)\|_{L^p(\mathbb{R}_+,\frac{{\rm d}t}{t})}.$$

In the second displayed formula in Section 10.6.b, '$\frac{1}{2\pi i}$' is missing in front of the integral.

page 451, line 7: Replace 'Letting' by 'letting'; line -10: replace 'C_0-semigroup' by 'bounded C_0-semigroup'.

page 453, in part (2), "$(S(t))_{t\in\mathbb{R}}$" should be replaced by "$(S(t))_{t\geq 0}$", in formula (10.45) "$g_f(t)$" should be replaced by "$g_f^+(t)$", and "$\|g_f\|_{L^1(\mathbb{R})}$" should be replaced by "$\|g_f^+\|_{L^1(\mathbb{R}_+)}$"; line 4 of the proof: "$\partial\Sigma_\nu$" should be replaced by "$\partial\Sigma_\nu^{\rm bi}$".

page 454, last formula in the proof of Proposition 10.7.2: In the first integral, "$g_f(t)$" should be replaced by "$g_f^+(t)$".

page 455, proof of Lemma 10.7.4: In the last displayed formula, the left-hand side should read "$|f'(z)|$".

page 458, Steps 1 and 3: In the displayed formulas, "$\|\kappa_a\|_{\mathscr{L}(\ell^p(\mathbb{Z}))}$" should be replaced by "$\|\kappa_a\|_{\mathscr{L}(\ell^p(\mathbb{Z};X))}$".

page 461, Theorem 10.7.10: in the first line of the proof, one needs to assume that $f \in H^1 \cap H^\infty$.

page 462, line 3 of Theorem 10.7.12: a period "." is missing; in the first line of the proof, one needs to assume that $f \in H^1 \cap H^\infty$; and in line 3 of the proof, a term $\|U\|_\infty^2$ is missing; in line 3 of Theorem 10.7.13, Replace "bounded analytic C_0-contraction semigroup' by 'bounded analytic C_0-semigroup".

page 464, statement of Lemma 10.7.16: In the last formula,

$$\mathscr{R}(\mathscr{T}) \leq \kappa_{2,p}^2 C^{1-p/2} \sup_{t>0} \|N(t)\|_p^{p/2}$$

should be replaced by

$$\mathscr{R}(\mathscr{T}) \leq \kappa_{2,p}^2 C^{1-p/2} \sup_{z\in\overline{\Sigma}_\delta} \|N(z)\|^{p/2}.$$

page 465, line 8: The line "with R-bound at most $k_{2,p}^2 C$" should be replaced by "with R-bound at most $k_{2,p}^2 C^{1-p/2} K^{p/2}$".

page 466, line 7: The inequality "$\|N(z)\|_{\mathscr{L}(S;\ell_k^q)} \leq K_0^{1-\theta} K_1^\theta$" should be replaced by "$\|N(z)\|_{\mathscr{L}(S;\ell_k^q)} \leq L_0^{1-\theta} L_1^\theta$"; line -5: "it satisfies $\|\psi(z)\|_{L^{p_j}(S;\ell_k^{q_j})} \leq L_j$ $(j = 0,1)$" should be replaced by "it satisfies $\|\psi(z)\|_{L^{p_j}(S;\ell_k^{q_j})} \leq L_j$ for $\Re z = j$ $(j = 0,1)$".

page 475, Theorem 10.8.1: A reference to Fendler's original paper is missing here and in the list of references: G. Fendler, Dilations of one parameter semigroups of positive contractions on L^p spaces, Canad. J. Math. 49 (1997), no. 4, 736–748.

page 476, displayed formula: Replace "$(D^\alpha x)(u)$" by "$(\partial^\alpha u)(x)$".

page 483, Problem P.9: In last line of the problem statement, "$\gamma_\infty(L^2(S);Y)$" should be replaced by "$\gamma(L^2(S);Y)$"; line 13: "$\gamma_\infty(L^2(S);Y) = \gamma_\infty(L^2(S);Y)$" should be replaced by "$\gamma_\infty(L^2(S);Y) = \gamma(L^2(S);Y)$".

page 484, line -15/16: Replace "is the generator semigroup" by "is the generator of a C_0-semigroup".

page 487, Appendix E, Remark $E.1.2$: In fourth line, "f" should be replaced by "ξ".

page 493, in the sentence before Theorem E.1.14, "almost surely" should be replaced by "in distribution".

page 496, formula (E.2): In the middle term in the integral, "$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$" should be replaced by "$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$".

page 515, in Definition F.3.2 add "for $p, q \in (0,\infty)$".

page 519, Lemma G.1.1: The correct bound reads '$\|R(\mu,A)\| \leq \frac{1}{1-\delta}\|R(\lambda,A)\|$ and is obtained by omitting the term '$-B^{-1}$' in the first line on the next page. With this term included, the argument shows that $\|R(\mu,A) - R(\lambda A)\| \leq \frac{\delta}{1-\delta}\|R(\lambda,A)\|$.

page 520, Lemma G.1.4: The 'arcsin' in the displayed formula should be replaced by 'sin'.

page 521, line -7: replace '$\|\!|$' by '$\|$'.

page 532, line 8: replace x by x^* in the first integral.

page 538, Proof of Theorem G.5.2 (2)⇒(1): The operators $S(t)$ should be defined through their representation as a Dunford integral provided in the statement of the theorem. The semigroup property is proved in the same way as the multiplicativity of the Dunford calculus; in line -8: replace 'a' by 'an'.

page 541, line 5: replace "z" by "ζ"; in Theorem G.5.3, one needs to add the assumption that S be bounded.

page 543, line 2: replace "$H^{2,p}(\mathbb{R};X)$" by "$H^{2,p}(\mathbb{R}^d;X)$".

page 553, Proposition H.2.5: "$H^1(\mathbb{S}_\vartheta)$" should be replaced by "$H^1(\Sigma_\vartheta)$".

page 571, middle of the proof: "$(M(T^j f))_N \leq T^j(M_N f)$" should be replaced by "$M_N(T^j f) \leq T^j(M_N f)$"; line -2: "$\xi_j = \mathbf{1}_{[-k-N,k+N]}(j)T^j f(s)$" should be replaced by "$\xi_j = \mathbf{1}_{[-k,k+N-1]}(j)T^j f(s)$".

page 572, Step 1: Missing period after "$T = (t_{i,j})_{i,j=1}^n$ on ℓ_n^p".

page 573, third formula: The middle term

$$\sup_{M\geq 1} \frac{1}{M} \sum_{m=0}^{M-1} \sum_{j\in J} (T^m \xi)_j \mathbf{1}_{N_j}$$

should be replaced by

$$\sup_{M\geq 1} \frac{1}{M} \sum_{m=0}^{M-1} \sum_{j=1}^{n} (T^m \xi)_j \mathbf{1}_{N_j}.$$

Step 2, third line: "E_π" should be replaced by "$\mathbb{E}_\pi$" twice.

page 574, third line before the end of the proof: "according to Step 3" should be replaced by "according to Step 2".

page 579, Theorem J.2.1: ϕ_{pr} and c_{pr} should be replaced by ϕ_{dpr} and c_{dpr}, respectively. (This is due to a related erratum in Proposition J.2.2 on page 580; see below.)

page 579, two lines before (J.4): "$\psi^{-r/p} w \in A_r$" should be replaced by "$\psi^{p-r} w \in A_r$".

page 580, claim (b) of Proposition J.2.2: c_p should be replaced by $c_{d,p} = c_d p'$. (The constant is produced as an application of Theorem J.1.1.) Several subsequent instances of c_p should be replaced accordingly.

Tuomas Hytönen, Jan van Neerven, Mark Veraar, Lutz Weis

(November 28, 2024)