Functional Analysis (corrected 2024 printing)

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Errata and Corrigenda (last update: September 29, 2025)

This list takes the corrected printing (dated October 2024 in the Preface), currently available online on the Cambridge website, as the point of departure. *Page and line numbers refer to this version of the book*. Some of these corrections have already been carried out in the current version on arXiv. A version of the book where all corrections have been carried out is downloadable by clicking on the book cover icon on the author's webpage; due to these corrections, page and line numbers may have changed in this version.

Errata, Typos, and trivia

page xii, line 14-15: replace 'domain f' by 'domain D'.
page 2, line -1: replace 'triangle inequality (ii)' by 'triangle inequality (iii)'.
page 9, line -2: replace 'K' by '$\mathbb{K}$'.
page 10, line 16: before 'an admissible constant' the word 'is' is missing.
page 15, line -1: Under the stated assumptions, $\|T\| = \max_{1\leqslant n\leqslant N}\|T_n\|$ may fail; it is correct under mild assumptions on the norm $|\cdot|$ on $\mathbb{K}^n$ in (1.1).
page 16, line line -9: For the equality $\|I_\mu = 1$ one needs that there exist a set $F \in \mathscr{F}$ with $0 < \mu(F) < \infty$ (otherwise $L^1(\Omega) = \{0\}$ and $\|I_\mu\| = 0$).
page 22, line 8: replace 'is contained in' by 'is a closed subset of'.
page 28, line 9: after 'let' insert a spacing.
page 36, line 8': replace 'satisfying' by 'for which'.
page 39, line 13: replace 'function' by 'bounded function'.
page 42, line -11: after 'an open set U in K' add 'containing x'.
page 48, line -9: in (2.4), replace '$u(t_{j,n})$' by '$u_n(t_{j,n})$';
line -6: replace '$u(t_{j,n})$' by '$u_n(t_{j,n})$'.
page 57, line 5: replace '$\int_D \big|\phi|_{U\setminus F}\big|^p \,\mathrm{d}x$' by '$\int_{U\setminus F} |\phi - \mathbf{1}_B|^p \,\mathrm{d}x$'.
page 59, line 3: replace '$\|g\|_p^{r-q}$' by '$\|g\|_q^{r-q}$'.
page 68, line 6: replace 'with $\mu(F') \geqslant 1$ and $\mu(F'') \geqslant 1$' by 'with $|\mu(F')| \geqslant 1$ and $|\mu(F'')| \geqslant 1$'.
page 70, line 18: replace '$\lim_{n\to\infty} k_n = 0$' by '$\lim_{n\to\infty} k_n = \infty$'.
page 73, linea -2 and -1: replace 'ν' and '$g \in L^1(\Omega,\nu)$' by 'μ' and '$g \in L^1(\Omega,\mu)$'.
page 78, line 14: replace '$\nu + (\mu - \nu)$' by '$\nu + (\mu - \nu)^+$'.
page 83, Problem 2.23: replace '(c)' and '(d)' by '(b)' and '(c)'.
page 92, line -12: replace 'a norm' by 'an inner product'.
page 109, Problem 3.4: Replace '$\|(1-\lambda)x + \lambda y\| < 1$' by '$\|(1-\lambda)x_0 + \lambda x_1\| < 1$'.
page 115, line -11: replace '$\ell \equiv 0$' by '$\phi \equiv 0$'.
page 121, line 14: a closing bracket is missing.

page 139, line -1: replace '$x, y \in X$' by '$x \in X$, $y \in Y$'.
page 154, line -9: after '$\|x^*_{n_k}\|$' add '$\|x\|$'.
page 161, line -1: we must assume $c_j \geqslant 0$ for all $j = 1, \dots, k$, and the same is true for the next line of the argument. At the end of the proof, consider positive and negative parts separately.
page 164, line -1: the index j_0 may depend on k, which creates a problem in the last line of the proof. A corrected proof is included in the version available on my homepage.
page 178, line 19: replace 'the proposition r gives that r establishes a isomorphism of' by 'Corollary 5.9 gives that r induces an isomorphism $r_/$ of'.
page 211, line -7: replace 'complex space' by 'complex Banach space';
line -6: replace 'will remain force' by 'will remain in force'.
page 214, line -12: replace '$U = (\mu - \lambda)I$' by '$U = (\lambda - \mu)I$'.
page 230, line -8: delete 'in K'.
page 232, line -8: replace '$\leqslant \lim_{j,k\to\infty} \|T\| \|y^*_{n_k} - y^*_{n_j}\|_\infty$' by '$= \lim_{j,k\to\infty} \|y^*_{n_k} - y^*_{n_j}\|_{C(K)}$'.
page 236, line -1: in the matrix, replace all occurrences of '$\mu - \lambda$' by '$\lambda - \mu$'.
page 237, line 3: replace '$\frac{1}{2\pi i} \int_\Gamma R(\mu, A)\, \mathrm{d}\lambda$' by '$\frac{1}{2\pi i} \int_\Gamma R(\mu, A)\, \mathrm{d}\mu$'.
page 239, line 3: replace 'with a closed subspace of X' by 'with a closed subspace of X^{**}.
page 243, line -2: delete 'orthogonal'.
page 249, line -8: replace '$T_\psi T_\phi$' by '$T_\phi T_\psi$'.
page 250, line -13: in the summation, replace '$j = (-n) \wedge 0$' by '$j = (-n) \vee 0$'.
page 257, Problem 7.20: instead of $z/(e^{kz} - 1)$, one should consider the function $z/(e^{z/k} - 1)$.
page 294, line 15: replace '$f_n - f_n$' by '$f_n - f_m$'.
page 311, line 3: replace '$F = \bigcup_{n\geqslant 1} F$' by '$F = \bigcup_{n\geqslant 1} F_n$'.
page 313, line 3: before 'the projection-valued ...' insert 'be'.
page 314, line 7: the condition '$0 \leqslant f_n \uparrow \mathrm{id}$' needs to be replaced by '$f_n \to \mathrm{id}$ uniformly'.
page 321, line -6: replace 'likely to arrive' by 'likely to arise'.
page 325, line 16: replace '$x \in B$' by '$x \in \mathsf{D}(B)$'.
page 332, line -2: replace '$\mu = -|\lambda|^2 / |\mathbb{R}e\lambda|$' by '$\mu = |\lambda|^2 / |\mathbb{R}e\lambda|$'.
page 341, line -11: replace '$\|x - P_{B_n} x\|$' by '$\|x - P_{B_n} x\|^2$'.
page 344, line -4: replace '$AT(^\star)^k$' by '$A(T^\star)^k$'.
page 350, line -3: replace '$\dots = g_z(A)(z - f(A)) = A$' by '$\dots = g_z(A)(z - f(A)) = I$'.
page 365, line 15: replace 'η_ε' by '$\eta_\varepsilon(x)$'.
page 367, line -13: replace '$\partial^k f_n^{(k)} \to g$' by '$\partial^k f_n \to g$';
line -10: replace g by g_U.
page 368, line 2: in the displayed equation, replace both g by ψ;
line 3: replace 'exists' by 'exist'.
page 370, line -10: replace '$\int_D (\rho \circ f)(\partial_j)\phi\, \mathrm{d}x$' by '$\int_D (\rho \circ f)\partial_j \phi\, \mathrm{d}x$'.
page 373, line -9: replace 'Proposition 11.1' by 'Proposition 11.10'.
page 374, line 6: replace '$n \geqslant 1$' by '$k \geqslant 1$';
line 13: replace '$\phi_n \in C_\mathrm{c}(\mathbb{R}^d_+)$' by '$\phi_n \in C^1_\mathrm{c}(\mathbb{R}^d_+)$'.
page 377, line -3: replace 'and we can find' by 'we can find'.
page 378, line -9: in the displayed formula in (ii), replace '$f \in W^{\ell,p}(\mathbb{R}^d)$' by '$f \in W^{\ell,p}(D)$';
line -3: to avoid confusion with the boundary regularity C^k, replace the indices k by indices m.
page 391, line -9: replace 'as $n \to \infty$' by 'as $k \to \infty$'.
page 398, line 1: replace '$\alpha \|\nabla v\|^2_{H^1_0(D)}$ by '$\alpha \|v\|^2_{H^1_0(D)}$.

page 402, Problem 11.23(a): replace '$\eta \in C_c^1(-1,1)$ satisfies $\eta(0) = 1$' by '$\eta \in C_c^1(0,\infty)$ satisfies '$\int_0^\infty \eta(r)\,\mathrm{d}r = 1$', and in the first displayed formula replace '$\eta'(y)$' by '$\eta'(r)$.
page 407, line 6: replace 'the' by 'a'.
page 411, line 7: replace '$u, y \in \mathsf{D}(\mathfrak{a})$' by '$x, y \in \mathsf{D}(\mathfrak{a})$'.
page 417, line 11: replace 'densely closed defined' by 'densely defined closed'.
page 417, line -5: replace 'in V' by 'in $\mathsf{D}(\mathfrak{a})$'.
page 422, line -9: replace '$\nabla f \in \mathsf{D}(\nabla)$' by '$\nabla f \in \mathsf{D}(\nabla^\star)$';
line -4: replace '$L^2(D)$' by '$L^2(\mathbb{R}^d)$'.
page 424, line -9: replace '$\frac{\partial f}{\partial \nu}\big|_{\partial D} = 0$' by '$\frac{\partial u}{\partial \nu}\big|_{\partial D} = 0$';
line -1: in equation (12.15), add the boundary condition '$\int_D \Delta u \cdot \phi\,\mathrm{d}x = -\int_D \nabla u \cdot \nabla\phi\,\mathrm{d}x$ for all $\phi \in H^1(D)$'. The same addition needs to be made line 2 on the next page.
page 427, line -10: replace '$-\Delta_{\mathrm{Neum}} u_n = -u_n'' = \pi^2 n^2 u_n$' by '$-\Delta_{\mathrm{Neum}} v_n = -v_n'' = \pi^2 n^2 v_n$'.
page 428, Lemma 12.25: assertion (2) does not require compactness and is an easy consequence of the identity $\frac{1}{\lambda-\mu} - R(\lambda,A) = \frac{1}{\lambda-\mu}(\mu - A)R(\lambda,A)$.
page 429, line 2: replace '$\sigma(R(\lambda,A)\setminus\{0\}$' by '$\sigma(R(\lambda,A))\setminus\{0\}$'.
page 431, line 13: replace '$L^2(\mathbb{R};\mathbb{C}^d)$' by '$L^2(D;\mathbb{C}^d)$'.
page 433, proof of Weyl's theorem: this proof can be shortened; see the downloadable version on my website.
page 442, Lemma 13.6 also works for intervals $[a,b]$ with $a < b$; this is used in the next lemma.
page 467, line 12: 'Step 1' can be removed.
page 516, line -5: replace 'Hilbert–Schmidt T operator' by 'Hilbert–Schmidt operator T';
line -2: replace '$\mathbb{C}^n$' by '$\mathbb{C}^d$'.
page 519, line 7: in the displayed formula, all four sums should all read '$\sum_{n\geqslant 1}$';
line -6: replace 'must by' by 'must be';
line -2: replace 'basis of T' by 'basis of H'.
page 548, in Problem 14.8, replace '$\|T\|_{\mathscr{L}_2(L^2(0,1))} \leqslant \|T\|$' by '$\|i\circ T\|_{\mathscr{L}_2(L^2(0,1))} \leqslant \|T\|$'.
page 552, first line of Section 15.1.b: replace 'be a measurable spaces' by 'be measurable spaces'.
page 555, line -9: replace '$\sum_{n=1}^N \leqslant 1$' by '$\sum_{n=1}^N c_n \leqslant 1$'.
page 556, line -12: replace '$P_\in \mathscr{P}_{\mathrm{fin}}(H)$' by '$P_n \in \mathscr{P}_{\mathrm{fin}}(H)$'.
page 558, line 4: before 'thereby completing' replace the dot by a comma;
line -10: replace 'step Step 5' by 'Step 5'.
page 560, line 8: replace 'ϕ' by 'ψ'.
page 562, line 15: replace 'Indeed, for' by 'Indeed, if'.
page 564, line 16-17: delete the second 'identify';
line -11: replace 'as an' by 'as a'.
page 570, line 12: after 'where' add '$I_0 := [0, \frac{1}{2^n}]$ and'.
page 572, line 18: replace 'proposition' by 'theorem'.
page 573, line 14: replace '$\mathscr{L}_1(T)$' by '$\mathscr{L}_1(H)$'.
page 582, line -1: replace 'thought as' by 'thought of as'.
page 585, line 1: replace 'of closed unit ball' by 'of the closed unit ball'.
page 614, line -12: at the end of the first line of this displayed formula, remove one closing bracket.
page 636, line 1: replace 'K' by '$\mathbb{K}$';
line 10: replace 'w' by 'W'.

page 641, line 8: replace ‘compact’ by ‘closed’;
line -11: replace ‘D’ by ‘X’.
page 645, line 11: delete ‘$x \in$’ twice; newline line -2: replace ‘π_i’ by ‘p_i’.
page 655, line 11: delete ‘for all $n \geqslant N$’.
page 655, line 6: replace ‘is measure’ by ‘is a measure’.
page 662, line 10: replace ‘in $\mathscr{M}_\mu$’ by ‘in $\mathscr{M}_\nu$’.
page 664, last paragraph of the proof of Theorem E.9: there is a small gap here: μ is defined on $\mathscr{R}$, but not necessarily $\mathscr{R}_n$ (we only know that $S_n \in \sigma(\mathscr{R})$. To fix this, assume that there exist $S_1, S_2, \cdots \in \mathscr{R}$ such that $S = \bigcup_{n \geqslant 1} S_n$ and $\mu(S_n) < \infty$ for all $n \geqslant 1$.
page 665, line 6: replace ‘$\beta_j - \alpha_j$’ by ‘$b_j - a_j$’;
line -7: replace ‘$\mu(A_n)$’ by ‘$\lambda(A_n)$’;
line -3: replace ‘Bolzano–Weierstrass’ by ‘Heine–Borel’.
page 666, line -3: replace ‘$\prod_{j=1}^n \mu_n$’ by ‘$\prod_{j=1}^n \mu_j$’.
page 678, Theorem F.9: The proof that $\mathscr{F}$ is a σ-algebra has a gap. Instead one can show that $\mathscr{F}$ is a Dynkin system, and then appeal to the proof of the Dynkin lemma to see that $\mathscr{F}$ equals the product σ-algebra $\mathscr{F} \times \mathscr{F}_2$.
page 684, line 13: replace ‘multipliers’ by ‘multiplier’.
page 687, line 2: after ‘Calkin’ add ‘algebra’;
line 6: replace ‘from $\mathscr{T}$’ by ‘from $\mathscr{T}/\mathscr{K}(H)$’;
line 8: repalce ‘$\mathscr{K}(H))$’ by ‘$\mathscr{K}(H)$’.
page 689, line 2: replace the second occurrence of ‘treatment’ by ‘theorem’.
page 690, line -5: replace ‘$\in L^2(D)$’ by ‘$\in L^2(\partial D)$’.
page 698, line 3: replace ‘C^{n+1}’ by ‘$\mathbb{C}^{n+1}$’.
page 699, line 8: replace ‘$n \in N$’ by ‘$n \in \mathbb{N}$’.

Addenda

page 89: Proposition 3.6 is a classical 1935 result of von Neumann and Jordan. It was pointed out by Heather MacBeth that the proof of the additivity of the inner product can be done in the following more compelling way. Consider the four instances of the parallelogram law:

$$\begin{aligned}
\|(x+y+z)+(x-z)\|^2+\|(x+y+z)-(x-z)\|^2 &= 2\big(\|x+y+z\|^2+\|x-z\|^2\big),\\
\|(x+y-z)+(x+z)\|^2+\|(x+y-z)-(x+z)\|^2 &= 2\big(\|x+y-z\|^2+\|x+z\|^2\big),\\
\|(y+z)+z\|^2+\|(y+z)-z\|^2 &= 2\big(\|y+z\|^2+\|z\|^2\big),\\
\|(y-z)+z\|^2+\|(y-z)-z\|^2 &= 2\big(\|y-z\|^2+\|z\|^2\big).
\end{aligned}$$

Taking suitable linear combinations of these four identities, we arrive at

$$\frac{1}{4}\Big(\|x+y+z\|^2-\|x+y-z\|^2\Big)=\frac{1}{4}\Big(\|x+z\|^2-\|x-z\|^2\Big)+\frac{1}{4}\Big(\|y+z\|^2-\|y-z\|^2\Big).$$

With the expression for the real part of the inner product (page 90, line 8), this reads

$$\Re(x+y|z)=\Re(x|z)+\Re(y|z).$$

The proof of the analogous identity for the imaginary parts can be done similarly.

page 298: One may observe that if T is normal and $f:\sigma(T)\to K$ is Borel measurable, where $K\subseteq\mathbb{C}$ is compact, then $\sigma(f(T))$ is contained in K. This follows by the same argument as in the first part of the proof of Proposition 8.23.

page 574, Theorem 15.32: Condition (i) can be equivalently expressed in terms of the POVMs $Q^{(i)}$ associated with the observables $\Phi^{(i)}$ as follows: the pure states $|\phi_1\rangle$ and $|\phi_2\rangle$ are indiscernible under $\Phi^{(i)}$, $i\in I$ if and only if for all $F\in\mathscr{F}$ we have

$$(Q_F h_1|h_1)=(Q_F h_2|h_2).$$

Expressing things this way, it becomes apparent that this is captures the underlying physics intuition correctly in case the POVMs $Q^{(i)}$ commute (in the sense that all projections involved commute). Indeed, since pure states are multiplicative (this is not proved here, but follows by combining 2.3.21 and 4.6.68 in volume 1 of Bratteli-Robinson, keeping in mind that our "pure states" correspond to "vector states" in this reference), the above condition self-improves to

$$(Ah_1|h_1)=(Ah_2|h_2)$$

for any finite product of the form $A=\prod_{j=1}^n Q_{F_j}^{(i_j)}$. In the case of non-commuting POVMs, this amounts to the physically 'correct' notion for indiscernibility.