Functional Analysis
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Errata and Corrigenda: arXiv version 5

The errata and typos of this list have been corrected in *arXiv version 5* of this manuscript. This list is now closed. A fresh list has be started which takes arXiv version 5 as the point of departure. A corrected author's version of the book is available by clicking on the book cover icon on the author's personal website.

Errata

page 29, Problem 1.10: in part (a), the condition $\|\cdot\| \leqslant C\|\cdot\|'$ should be added.
page 30, Problem 1.15: replace 'is a proper dense subspace of $C[0,1]$' by 'is not closed'.
page 150, line -3: the equality $\langle x_0^*, F\rangle = \overline{B}_{\mathbb{K}}$ follows from the Banach–Alaoglu theorem in the next section. Indeed, by that theorem, F is weak* compact in X^{**} and therefore its image under x_0^* is compact, hence closed, in $\mathbb{K}$. On the other hand, from $B_X \subseteq F$ it follows that $\langle x_0^*, F\rangle \supseteq B_{\mathbb{K}}$.
page 160, lines 6–7: the functions $\mathbf{1}_K$ do not belong to $C(K_j)$; instead, the bound $\nu_j(K_j) \geqslant 1-2^{-j}$ follows from Proposition 4.65.
page 226, Problem 6.15: the condition '$\sigma(T) = \{1\}$' should be added.
page 234, line -3: one should sum over the dimensions of all Jordan blocks corresponding to λ.
page 295, lines 15–16: delete the sentence 'By ... $\sigma(T)$'.
page 296, lines 14–17: instead of commutation, use the identity $P_B = P_{U_n} - P_{U_n\setminus B}$ and the preceding estimate to obtain the bound $|(P_B^2 - P_B)x|x)| \leqslant \frac{5}{\sqrt{n}}\|x\|$.
page 299, proof of Theorem 9.20: this appears to be (part of) a proof that has been replaced by the one of Proposition 9.9. Applying this proposition with $g(\lambda) = \lambda$ and using Theorem 9.13, Theorem 9.20 immediately follows.
page 375, proof of Theorem 11.34: the proof contains a small computational error. Eliminating it results in the constant $C_{p,D} = 2rp^{-1/p}$. More generally, the argument works for open sets D contained in $(-r,r)\times\mathbb{R}^{d-1}$.
page 389, Problem 11.8: delete parts (a) and (b), and reformulate the hint in (c) as 'First prove that $\psi f \in H^1(D)$ for every test function $\psi \in C_c^\infty(D)$. Then use Lemma 11.36.'
page 414, line -1: not the Laplacians themselves, but their negatives are positive.
page 434, proof of Theorem 13.11: the first step of this proof works if every weak orbit $t \mapsto \langle S(t)x, x^*\rangle$ is assumed to be continuous for $t \geqslant 0$. Unter the assumptions as stated, the local boundedness can be obtained by arguing as in Proposition 13.3.

page 461, Lemma 13.33: replace the part of the statement starting with ‘the open ...’ by: for all $\varphi \in (0, \frac{1}{2}\pi)$ with $\sin\varphi < 1/M$ we have $\Sigma_\varphi \subseteq \rho(A)$ and

$$\sup_{\lambda\in\Sigma_\varphi} \|\lambda R(\lambda,A)\| \leqslant \frac{M}{1-M\sin\varphi}.$$

This is also what is actually being proved.

page 464, lines 11–16: replace the text starting at ‘By Lemma ...’ by ‘A similar argument shows that for all $0 < \theta < \frac{1}{2}\pi + \eta$ we have $\Sigma_\theta \subseteq \rho(A)$ and $\|R(\lambda,A)\| \leqslant M|\lambda|^{-1}$ for some constant $M \geqslant 0$ depending on θ. Theorem 13.30 then implies that the semigroup generated by A is bounded analytic on every sector $\Sigma_{\eta'}$ with $0 < \eta' < \eta$. Its contractivity on these sectors is obtained by applying the Hille–Yosida theorem to the operators $e^{i\eta'}A$’.

page 465, part (1) of Theorem 13.37: $-A$ should be as in Corollary 12.13, that is, $-A$ is associated with a densely defined closed continuous accretive form in H.

page 481, Theorem 13.51: the proofs of L^∞-boundedness and weak* continuity assume selfadjointness. This additional assumption is satisfied in the subsequent application, but can be avoided as follows. For all $f \in L^2(\Omega)$ and $g \in L^\infty(\Omega)$ we have

$$|\langle f, Tg\rangle| \leqslant \langle |f|, T|g|\rangle \leqslant \langle |f|, T\mathbf{1}\rangle \|g\|_\infty \leqslant \langle |f|, \mathbf{1}\rangle \|g\|_\infty = \|f\|_1 \|g\|_\infty.$$

Since $L^2(\Omega)$ is dense in $L^1(\Omega)$, it follows that $\|Tg\|_\infty \leqslant \|g\|_\infty$. It follows that part (1) holds for $1 \leqslant p \leqslant \infty$. Part (2) can be omitted.

page 518, Proposition 14.21: the proof contains a gap. It is easily corrected by using the fact (see Dunford & Schwartz II, Lemma XI.6.21) that if S is a linear operator on an N-dimensional Hilbert space H_N with eigenvalue sequence $(\lambda_n)_{n=1}^N$ repeated according to multiplicities, there exists an orthonormal basis $(h_n)_{n=1}^N$ in H_N such that $(Sh_n|h_n) = \lambda_n$ for all $n = 1, \dots, N$.

page 526, line -2: replace ‘If T is ... we find that’ by ‘If T is a linear operator acting on $\mathbb{C}^d$, the result of Step 1 of the proof of Proposition 14.21 shows that’.

page 573, Theorem 15.31: the formula in the theorem should read $U(t)\Phi_B U^\star(t) = \Phi_{e^{-it}B}$ (there a sign error in the calculation).

page 574, Theorem 15.32: Carefully writing out the ‘topological subtleties’ referred to in the proof, one finds that Ω should be assumed to be a locally compact Hausdorff space whose topology is countably generated, and the space X constructed in the proof is in general only locally compact. The formulation of the theorem also contains an editing error, which is corrected below.

page 596, Theorem 15.55: ‘$W(a_s)$’ should be replaced by ‘$W(\widehat{a_s})$’. To derive the first displayed formula in the proof, one begins by writing out the definition of the Fourier transform. This leads to the corrected expression

$$W(\widehat{a})f(y) = \int_{\mathbb{R}^{2d}} a\Big(\xi, \frac{1}{2}(x+y)\Big) \exp(i\xi(x-y)) f(x)\,\mathrm{d}m(x)\,\mathrm{d}m(\xi),$$

which can be used to complete the proof.

page 599, line -8ff: replace ‘$h \in \mathbb{K}^d$’ by ‘$h \in \mathbb{R}^d$’ throughout the rest of the chapter; the spaces $L^2(\mathbb{R}^d, \gamma)$ can be taken real or complex.

page 635, line 10: replace ‘$x(i) \in p_i(M) \cap U_i$ and hence $x \in M \cap p_i^{-1}(U_i)$’ by ‘$p_i(M) \cap U_i \neq \varnothing$ and hence $M \cap p_i^{-1}(U_i) \neq \varnothing$’; in line -11 replace ‘$\cap\cap$’ by ‘$\cap$’.

Typos and other trivia

page 6, line 6: replace '$\leqslant \|[x_n]\|$' by '$\leqslant \|[x_n]\| + \frac{1}{n^2}$'
page 7, line 9: replace 'the next section' by 'Section 1.3'.
page 8, line 7: replace '$\|f_n - f\|_\infty = 0$' by '$\lim_{n\to\infty} \|f_n - f\|_\infty = 0$'.
page 9, line -10: after the first 'in' add 'X'; 'if' and 'only if' should be interchanged.
page 10, line 13: replace 'contants' by 'constants'.
page 11, line 9: replace '$T, T \in \mathscr{L}(X,Y)$' by '$T, T' \in \mathscr{L}(X,Y)$'.
page 14, line -6: here we use the notation $\langle y, y^* \rangle := y^*(y)$; line -4: uniqueness of weak limits is assured by the Hahn–Banach theorem (see Corollary 4.11).
page 16, line -8: replace 'T_n' by 'T_m'; line -3: replace 'obtain' by 'obtained'.
page 17, Example 1.31: in fact the last part of example 1.30 gives $\|T\| \leqslant 1/\sqrt{2}$.
page 21, line 7: replace '$\|x_j - x_k\| \geqslant 1$' by '$\|x_j - x_k\| \geqslant \frac{1}{2}$'.
page 32, Problem 1.26: replace 'almost all' by 'all' (at least for part (a)).
page 37, line 8: replace '$\leqslant \frac{2}{\delta^2 n}\|f\|_\infty$' by '$\leqslant \varepsilon + \frac{2}{\delta^2 n}\|f\|_\infty$'
page 40, line 3: after 'exists' add 'a'; line 12: replace '*pi*' by 'π'.
page 44, line -4: replace 'differentiable' by 'continuously differentiable'.
page 46, line -1: replace '$f(t,y)$' by '$f(s,y)$'; line -11: replace '$j = 1,\dots,2^n$' by '$j = 0,\dots,2^n - 1$'; line -4: one comma.
page 53, line 1: replace '$< \infty$' by '$= 0$'.
page 55, line 10: omit 'whose ... complement'.
page 59, line 6: replace '$x - h$' by '$x + h$' ($2\times$); lines 8–10: replace '$-Nh$' by 'Nh' ($3\times$).
page 63, line 14: delete 'a'.
page 66, line 7: replace 'A'' and 'A''' by 'F'' and 'F''', respectively.
page 72, line -13: replace '$\{f \leqslant 0\}$' by '$\{f < 0\}$'; line -8: replace '$\nu(F)$' by $|\nu|(F)$'.
page 75, line 5: the decomposition is not unique, but has a minimality property discussed in Problem 2.42; line 16: replace '$u \vee v$' by '$v \vee w$'; line -18: delete '$\leqslant u$'; line -5: replace 'vector space' by 'normed space'.
page 76, line -11: add 'is'.
page 77, Problem 2.1: replace '$\|a\|_p \leqslant \|a\|_r$' by '$\|a\|_r \leqslant \|a\|_p$'.
page 79, Problem 2.10: in (b), replace both 'A' by 'F'.
page 79, Problem 2.42: in (c), replace '$w \perp w'$ by '$w \wedge w' = 0$'.
page 96, line 15: replace '$(y_j|x_n^{(2)})$' by '$(y_j|x_n^{(k)})$'; line -3: replace the first '$= 0 =$' by '$=$'.
page 100, line -8: replace '$[\pi,\pi]$' by '$[-\pi,\pi]$'.
page 106, line 13: replace '$L^2(K_j,\mu_k)$' by '$L^2(K_j,\mu_j)$'.
page 118, line 10: replace 'it is was shown there' by 'it was shown'.
page 119, line 9: replace 'X' by 'K'; line 12: one should observe that the argument of Step 4 holds for any representing Radon measure μ; line -8: this argument is somewhat obscure. For a cleaner version replace 'Choose functions ... desired properties.' by 'For every x in the compact support of f choose an open set V_x with compact closure such that $x \in V_x$. By compactness, the support of f is contained in a finite union $V := \bigcup_{m=1}^M V_{x_m}$. For $j = 1,\dots,n$ set $V_j := U_j \cap V$. Then $\mathrm{supp}(f)$ is contained in $\bigcup_{j=1}^n V_j$, and this union has compact closure. Hence we may use Theorem C.11 to select a partition of unity $(g_j)_{j=1}^n$ relative to the sets V_j, $j = 1,\dots,n$, such that $\sum_{j=1}^n g_j \equiv 1$ on $\mathrm{supp}(f)$. Then $g_j \in C_c(X)$ and $g_j \prec V_j$ for $j = 1,\dots,n$; from $V_j \subseteq U_j$ we infer that also $g_j \prec U_j$.'
page 122, line 11: replace 'Borel B sets' by 'Borel sets B'; line -13, -14: one could replace '2ε' and '3ε' by 'ε' and '2ε', respectively.

page 123, line 16: add 'with $f_1, f_2 \geqslant 0$'.
page 125, line 1: one should observe that the identity $\|g\|_q = \|\phi\|$ holds for any representing function $g \in L^q(\Omega)$ (by Hölder's inequality and Proposition 2.26); line 14: replace '$\{g \leqslant n\}$' by '$\{|g| \leqslant n\}$'.
page 127, line 15: replace the first '$u' \in [0,x']$' by '$u \in [0,x]$'.
page 133, line 11: delete the second 'Y'; proof of (2): Since finite-dimensional subspaces are closed, this is immediate from the definitions.
page 139, line 9: replace 'T_{k*}' by 'T_{k^*}'.
page 143, line -10: replace the first '$\langle x_0, x^* \rangle$' by '$\langle x, x^* \rangle$'.
page 150, line 1: replace 'contracting' by 'contradicting'.
page 152, line 3: replace 'lemma' by 'theorem'.
page 157, lines 1–7: replace the proof by the observation that the lemma follows from the uniqueness part of Theorem 4.2, viewing $g\,\mathrm{d}x$ as a finite Borel measure on $\mathbb{R}^d$; line 11: replace '$f \in L^1(\mathbb{R}^d)$' by '$f \in C_{\mathrm{c}}(\mathbb{R}^d)$; line 12: replace 'almost all' by 'all'; line -5: add 'for all $f \in C_{\mathrm{c}}(\mathbb{R}^d)$'.
page 158, line 6: replace '$Tf(y)$' by '$Tf(x)$', and add that the identity holds for all $x \in \mathbb{R}^d$ by a continuity argument; line 9: replace '$f * \mu = 0$ for all $f \in L^1(\mathbb{R}^d)$' by '$\int_{\mathbb{R}^d} f(x-y)\,\mathrm{d}\mu(y) = 0$ for all $f \in C_{\mathrm{c}}(\mathbb{R}^d)$ and $x \in \mathbb{R}^d$'; line 10: replace '$(f * \mu)(x)$' by '$\int_{\mathbb{R}^d} f(x-y)\,\mathrm{d}\mu(y)$'; lines 12–13: replace 'for almost all $x \in \mathbb{R}^d$ ... was arbitrary' by 'for all $x \in \mathbb{R}^d$'.
page 160, line -3: replace 'for $m \geqslant m_\varepsilon$ (with m_ε as in Step 2)' by 'for $m \geqslant 1$ large enough'.
page 165, Problem 4.10: the text after (b) should appear before (a).
page 167, Problem 4.18: this problem is solved in the main text. Solve the analogous problem for $L^2(0,1)$.
page 169, Problem 4.34: in (a), replace '$|f_n| \varepsilon k$' by '$|f_n| \leqslant k$'.
page 173, line 4: completeness of Y is not needed; line 6: replace 'linear bounded' by 'linear and bounded'; line -1: replace '4.7.b' by '4.2'.
page 176, line -11: replace '1.32' by '1.33'.
page 181, line 6: replace '5.4' by '(5.4)'.
page 182, line 9: replace '$g^{(\lambda)}$' by '$g^{(\lambda)}(x)$'.
page 187, line 6–12: replace this by 'To prove that $\mu = 0$, by the uniqueness part of the Riesz representation theorem it suffices to show that $\int_{\mathbb{R}^d} f\,\mathrm{d}\mu = 0$ for all $f \in C_{\mathrm{c}}(\mathbb{R}^d)$'.
page 188, line -11: replace 'measure space' by 'σ-finite measure space'.
page 197, line -1: replace '$I + x$' by '$I - x$'.
page 198, line 11: after 'vanishes' add 'on $\mathbb{R}_-$'.
page 199, line -14 and -12: replace 'L^p' by L^{2p}' (3×); line -3: replace '$\|f\|_q$' by '$\|f\|_p$'.
page 201, line -2 of Problem 5.3: change '$\|T_{i_n} x_n\| \geqslant n$' to '$\|T_{i_n} x\| \geqslant n$'.
page 202, Problem 5.8: replace '$(x_n)_{n \geqslant 2}$' by '$(x_n)_{n \geqslant 1}$'.
page 207, Problem 5.23: in parts (a) and (b), replace 'P_y' by 'p_y'.
page 214, lines 15–16: delete 'on $L^2(\mathbb{T})$'.
page 216, line 20: replace '$\partial\sigma_A(T)$' by '$\partial\sigma_{\mathscr{A}}(T)$'.
page 219, line 5: replace '$n \in N$' by '$n \in \mathbb{N}$'.
page 221, proof of Theorem 6.23: the first part of the proof can be omitted if one takes the limit superior (instead of the limit) at the end of the second part of the proof.
page 224, Problem 6.2: replace 'r' by '$\|R(\lambda, T)\|^{-1}$'.
page 229, line -5: replace the first 'μ' by '$\mu \times \mu$'.
page 231, line 11: replace '$\|x\|$' by '$\|x + Y\|$'.
page 246, line 10: replace '$\frac{1}{2\pi}$' by '$\frac{1}{2\pi i}$' in the second integral.

page 248, line -7: insert a constant C' in front of the third term on the left-hand side.
page 253, Problem 7.8: replace 'use the result of the preceding problem' by 'argue as in Problem 7.3'; Problem 7.11: replace '$X_{\{\lambda\}}$' by 'X_λ in (a) and (b), and replace (c) by: 'deduce that $X_\lambda = \mathsf{N}(\lambda - T)^\nu$'.
page 256, line 10: replace 'and the real numbers, the positive real numbers' by 'the complex numbers'.
page 270, line -5: replace 'op' by 'of'.
page 271, line 13: replace 'Theorem 8.20' by 'Corollary 8.24'.
page 274, line -10: replace 'f_h' by '$f_h + N$'.
page 275, line 9: replace 'representation' by 'unitary representation'; line 12: replace the first 'U' by '$\widetilde{U}$'.
page 276, line -11: replace '$\widetilde{T}^k h_k$' by '$\widetilde{T}^k J h_k$'.
page 279, Problem 8.14: delete the dot in the displayed formula; Problem 8.15: replace the first 'space' by 'spaces'.
page 283, line 2: replace 'an' by 'and'; line 4: replace '$h \bar{\otimes} k$' by '$k \bar{\otimes} h$'; line 7: replace 'basis' by 'bases'.
page 291, line -9: replace '$\mathbf{1}_{B_n(\lambda)}$' by '$(P_{B_n} x_n | x_n)$'.
page 295, line 6: delete '$x = y$,' and move 'and $P_x := P_{x,x}$' to the next line.
page 296, line 10: replace 'we have' by a comma; line 18: this has already been used earlier in the proof; more generally, on the previous page one could have observed that $\|f(T)\| \leqslant \|f\|_\infty$ for all $f \in B_\mathrm{b}(\sigma(T))$.
page 297, line 10: replace 'f_n' by 'f'.
page 300, line 5: replace 'Theorem 9.8' by 'Theorem 8.22'.
page 301, line -9: replace '$f_1(P_x)$' by '$f_1(Q_x)$'.
page 326, line -12: replace '$\mathsf{R}(A+i) = \mathsf{R}(A^\star + i)$' by '$\mathsf{R}(A-i) = H$; line -9: replace 'the assumptions imply' by 'the assumption $\mathsf{R}(A+i) = H$ implies'.
page 338, line 5: replace '$\widetilde{Q}_x$' by '$\widetilde{Q}$'; lines 8, 11, 13: replace 'Ψ' by 'Φ'.
page 340, line -3: replace 'with ... proof' by 'where the penultimate identity is proved as in Example 4.9'.
page 342, line -4: replace 'selfadjoint' by 'normal'.
page 343, line -7: The first identity follows by repeating the preceding argument with f replaced by $f - p_n$.
page 348, line 3: replace 'C^k' by '$C^{|\alpha|}$'; lines -8 and -7: this sentence should be deleted.
page 349, line 1–3: instead of 2δ one could just take δ.
page 352, line 6: there is no need of a subsequence argument.
page 353, line -6: replace 'g' by 'η' ($3\times$).
page 354, lines 3–5: replace '$\partial_x^\alpha f$' (twice) and '$\partial_y^\alpha f$' by '$(\partial^\alpha f)$'; line 4: the big closing bracket is in the wrong place; lines 5–7: the term '$(-1)^{|\alpha|}$' is missing ($3\times$); lines 8–9: delete 'Here ... taken'; line 12: the closing dot is missing.
page 355, line 2: after 'of order α' add 'in $L^p_{\mathrm{loc}}(D)$'; second and third line of the 'only if' proof: replace '$C_\mathrm{c}(D)$' by '$C_\mathrm{c}^\infty(D)$'.
page 356, line 14: replace '$C_\mathrm{c}^2(\mathbb{R})$' by '$C_\mathrm{c}^k(\mathbb{R})$'; line 17: replace '$\zeta'(0) = \zeta''(0) = 0$' by $\zeta'(0) = \cdots = \zeta^{(k)}(0) = 0$'.
page 357, line 6: add '$\mathrm{d}t$' in the integral; line -1: after 'so does' add 'the'.
page 358, line 8: replace 'For' by 'Fix'.
page 360, line 3: replace '$f_n^{(\alpha)}$' by '$\partial^\alpha f_n$'.
page 361, line 7: replace 'belongs to' by 'is weakly differentiable'.

page 364, line -9: replace ‘$\partial_d(x',y)$’ by ‘$\partial_d\phi_n(x',y)$’; line -6: replace ‘$\int_0^{x_d}$’ by ‘$\int_0^{\varepsilon}$’.
page 365, line 16: replace ‘obtain’ by ‘obtains’; line -2: after ‘and’ add ‘in this case there exists a constant $C > 0$ such that’.
page 367, line -3: replace ‘$\bigcup_{m=1}^M$’ by ‘$\bigcup_{m=0}^M$’.
page 368, line -8: replace ‘C_ℓ’ by ‘C’.
page 370, line -11: after ‘Moreover,’ add: ‘$\mathsf{D}(\Delta) = W^{2,2}(\mathbb{R}^d)$ and’.
page 373, line -9: omit ‘in’.
page 375, line 2: replace ‘side’ by ‘sides’.
page 376, line 1: replace ‘$\|u\|_{H_0^1(D)}$’ by ‘$|||u|||_{H_0^1(D)}$’; line -9: replace the second ‘$L^2(D)$’ by ‘$L^2_{\rm loc}(D)$; line -5 to -1: delete ‘In particular … gives the result’.
page 377, line 6: replace ‘$H^2_{\rm loc}(D)$’ by ‘$H^2(D)$’; replace ‘The proof … work’ by ‘This follows from Theorem 11.28 and Lemma 11.36’.
page 380, line 2: after ‘from’ add ‘it’; line -6: replace ‘$= |h|^p\|f\|_{W_0^1(D)}$’ by ‘$\leqslant |h|^p\|f\|^p_{W_0^1(D)}$’.
page 382, line -11: replace ‘$\|u\|_{H^1_{\rm av}(D)}$’ by ‘$|||u|||_{H^1_{\rm av}(D)}$’.
page 383, line 4: omit ‘$\in$’.
page 387, line -6 and further: replace ‘$\overline{a}$’ by ‘$a^\star$’, where $a^\star_{ij} = \overline{a_{ji}}$.
page 388, Problem 11.2(c): replace ‘$W_0(0,1)$’ by ‘$W_0^{1,p}(0,1)$’; remove ‘(see Problem 11.2)’ .
page 389, Problem 11.7: in the first line, after ‘∂’ add ‘as’; in the second, replace ‘∂^α’ by ∂’.
page 390, Problem 11.14(c): replace ‘Cauchy’ by ‘bounded’; Problem 11.16: replace ‘$\|f\|_{W^{1,\infty}(D)}$’ by ‘$\|\nabla f\|_{L^\infty(D;\mathbb{K}^d)}$’ (three times).
page 391, Problem 11.19(b): replace ‘we have $1 < p < \infty$, then’ by ‘with $1 < p < \infty$ we have’.
page 400, line -2: replace ‘$\|u\|\|v\|$’ by ‘$\|u\|_V\|v\|_V$’.
page 401, line 14: replace ‘$\mathsf{D}(A)$’ by ‘$\mathsf{D}(\mathfrak{a})$’.
page 406, line 14: in additions to the stated assumptions, $\mathfrak{a}$ should be densely defined; line -5: replace ‘accretivity constant’ by ‘coercivity constant’.
page 409, line 9: replace ‘because of the restatement of (2) mentioned at the beginning of the proof’ by ‘because $(u_n - u'_n)_{n\geqslant 1}$ is an approximating sequence for 0, so $(2')$ can be applied with v_n replaced by $u_n - u'_n$; to see this, note that

$$\begin{aligned}
&\Re\mathfrak{a}((u_m-u'_m)-(u_n-u'_n),(u_m-u'_m)-(u_n-u'_n)) \\
&\quad = \Re\mathfrak{a}((u_m-u_n)-(u'_m-u'_n),(u_m-u_n)-(u'_m-u'_n)) \\
&\quad \leqslant C\|u_m-u_n\|^2_{\mathfrak{a}} + 2C\|u_m-u_n\|_{\mathfrak{a}}\|u'_m-u'_n\|_{\mathfrak{a}} + C\|u'_m-u'_n\|^2_{\mathfrak{a}}
\end{aligned}$$

by the continuity of $\mathfrak{a}$, and all three terms on the right-hand side tend to 0 and $m,n\to\infty$ since both $(u_n)_{n\geqslant 1}$ and $(u'_n)_{n\geqslant 1}$ are approximating sequences for $\overline{u}$, and therefore Cauchy with respect to $\|\cdot\|_{\mathfrak{a}}$’.
page 413, line -3: replace ‘Δf’ by ‘Δu’.
page 418, line 4: replace ‘$\Delta u = f$’ by ‘$-\Delta u = f$’.
page 420, line 2: add the condition that D be connected; line 13: replace ‘$j = 0,\dots,n$’ by $j = 1,\dots,n$’.
page 421, line 10: replace ‘By (12.16) this implies $f_{n_k} \to f$ weakly in $L^2(D)$’ by ‘Since bounded operators are weakly continuous and ∇ is bounded from $H_0^1(D)$ to $L^2(R;\mathbb{C}^d)$, this implies $\nabla f_{n_k} \to \nabla f$ weakly in $L^2(D;\mathbb{C}^d)$’.
page 424, lines 9–12: replace ‘$(2r)^d$’ by ‘$(2\rho)^d$’ (four times).
page 425, Problem 12.2(c): replace ‘$+(u|v)$’ by ‘$+\lambda(u|v)$’; Problem 12.4: replace the second ‘b’ by ‘c’.
page 426, Problem 12.7: replace ‘$\mathfrak{a}$’ by ‘Re$\mathfrak{a}$’ (twice).

page 427, line 10: replace ‘Linear equations’ by ‘Equations’.
page 434, line -12: replace ‘$\Re\lambda \geqslant \omega + 1$’ by ‘$\lambda \geqslant \omega + 1$’; line -9: replace ‘$S(t))_{t\geqslant 0}$’ by ‘$(S(t))_{t\geqslant 0}$’.
page 435, line 13: replace ‘on X’ by ‘on x’.
page 436, line 10: replace ‘$A = A_-$’ by ‘$A = -A_-$’; line 15: replace ‘$S(-t)$’ by $S_-(t)$’.
page 442, line -3: the term ‘s^n’ should be placed inside the integral.
page 445, lines 11–14 should be deleted; this is an editing mistake.
page 446, line -10: an x is missing behind the term ‘$(I - e^{-\nu t}S(t))$’.
page 453, line -10: remove ‘fixed point!argument’; this is an editing mistake.
page 457, line -9: replace ‘$(0, \frac{1}{2}\pi)$’ by ‘$(\frac{1}{2}\pi, \theta)$’; line -8: replace ‘$|\arg(z)|$’ by ‘$|\arg(\zeta)|$’; line -3: replace ‘right’ by left’.
page 458, line 3: the last double integral in this computation should be replaced by the single integral over Γ'; line 6: replace ‘$|\arg(z)|$’ by ‘$|\arg(\zeta)|$’.
page 459, line 4: replace ‘$\frac{1}{2}\pi - \theta$’ by ‘$\theta - \frac{1}{2}\pi$’; line 6: replace ‘$\mu \in \Sigma_\eta \setminus \Sigma_{\eta'}$’ by ‘$\mu \in \Sigma_\theta \setminus \Sigma_{\theta'}$, where $\frac{1}{2}\pi + |\arg(\zeta)| < \theta' < \theta$ as before’.
page 464, line 9: replace ‘$\ldots \leqslant |\lambda_t| \leqslant 1 + |\lambda_1|$’ by ‘$\ldots \leqslant |\lambda_t|^{-1} \leqslant m^{-1}$, where $m = \min_{0\leqslant t\leqslant 1} |(1-t) + t\lambda_1|$’; line 11: replace ‘$|\lambda|$’ by ‘$|\lambda|^{-1}$’.
page 466, line 10: replace ‘for’ by ‘form’; in Proposition 13.42 and its proof, all $1/\tan\omega$ should read $\tan\omega$.
page 468, line -15: replace ‘u’ by ‘u'’.
page 469, line -11: interchange ‘$+$’ and ‘$=$’.
page 470, line 13: here and in the rest of the proof, replace ‘$C_c(\mathbb{R})$’ by ‘$C_c(\mathbb{R}_+)$’.
page 471, line 2: replace ‘$f(t)$’ by ‘f’; line -6: replace ‘$S(t)$’ by ‘$S(t-s)$’.
page 472, line 11: replace ‘$T_{m_r}f \to T_m$’ by ‘$T_{m_r}f \to T_m f$’.
page 473, line 11: replace ‘$U(t)x$’ by ‘$U(\pm t)x$’.
page 477, line 13: replace ‘$e^{-x^2/4t}$’ by ‘$e^{-|x|^2/4t}$’.
page 478, line 13: density $C_c^\infty(\mathbb{R}^d)$ in $\mathsf{D}(\Delta)$ was not mentioned in Section 11.1.e, but is easy to prove: first, by a mollification argument, $C^\infty(\mathbb{R}^d) \cap \mathsf{D}(\Delta)$ is dense in $\mathsf{D}(\Delta)$, and the asserted density follows by a truncation argument.
page 480, line -12: replace ‘A’ by ‘$-A$’; line -6: replace ‘In Section 13.6.c’ by ‘Above’.
page 483, line 14: replace the first ‘$u - v$’ by ‘$v - u$’.
page 484, lines -12, -8, -7 (first integral): a factor ‘$\frac{1}{\pi}$’ is missing in front of the integrals (3 times); lines -4, -3, -2: ‘x’ and ‘ξ’ should be interchanged; line -1: omit ‘$\frac{1}{(2\pi)^d}$’.
page 490, line -5: replace ‘Corollary 15.59’ by ‘Proposition 10.32’; line -2: replace ‘extend’ by ‘extends’.
page 491, line 14: the form is also continuous, and A is selfadjoint (no closure needed); line -15: omit ‘$H =$’; line -9: omit ‘(a rescaled version of)’.
page 492, line 4: there is a bracket size mismatch in the displayed formula; line -3: replace ‘$\neq 0$’ by ‘$t \neq 0$’.
page 494, line 10: the term ‘$u_2\overline{v_1}$’ should be deleted from the integrand.
page 496, line -5: the first norm should be the $H^1(\mathbb{R}^d)$-norm.
page 497, line 13: replace ‘$\lim_{t\to 0} m_{1,1;t}\widehat{f} = f$’ by ‘$\lim_{t\to 0} m_{1,1;t}\widehat{f} = \widehat{f}$’.
page 500, line 2: replace ‘$H(t)$’ by ‘$P(t)$’ (and observe a slight abuse of notation).
page 501, Problem 13.3(d): replace ‘$nR(n,A)A$’ by ‘$nAR(n,A)$’.
page 503, Problem 13.9: delete ‘the symmetric matrix Q is positive and conclude that’; Problem 13.14 is in fact the preamble to this problem.
page 506, Problem 13.22: delete ‘for all $\varepsilon > 0$’.

page 510, line -12: replace 'H' by '$L^2(\Omega,\mu)$'.
page 511, line 4: the inequality needs justification. The issue is avoided by arguing as follows: By the result already proved for Hilbert–Schmidt operators,

$$\sum_{n\geqslant 1}(Th'_n|h'_n) = \sum_{n\geqslant 1}\|T^{1/2}h'_n\|^2 = \sum_{n\geqslant 1}\|T^{1/2}h_n\|^2 = \sum_{n\geqslant 1}(Th_n|h_n).$$

page 513, line 5: replace 'bases' by 'sequences'. The same change has to be made in line -10 and in line -1 on the next page.
page 514, line 7: the triangle inequality for $\mathscr{L}_1(H)$ hasn't been proved yet at this point, but us not needed here: since $(g_n)_{n\geqslant 1}$ and $(h_n)_{\geqslant \text{`}}$ are orthogonal sequences, the trace can be evaluated directly to be $\sum_{n\geqslant N+1}\lambda_n$ (no absolute value needed since the λ_n are positive); line 12: replace '$Th=0$ for $h\in Y_0 := \mathsf{N}(T)$' by '$|T|h=0$ for $h\in Y_0 := \mathsf{N}(|T|)$'; line 14: one should test against an orthonormal basis $(h'_m)_{m\geqslant 1}$ containing $(h_n)_{n\geqslant 1}$ as a subsequence.
page 515, line -1: replace '$(Th'_n|h_k)(h_k|h'_n)$' by '$(Th_k|h'_n)(h'_n|h_k)$' twice, and make the corresponding adjustments in the next three lines.
page 516, line -2: replace 'and $P^2h_n = \cdots = 1$' by '$P^2h_n = \|h_n\|^2\sum_{m=1}^N (g_n|h_m)g_m$ we deduce that $(g_n|h_m) = \delta_{mn}$'.
page 517, line -4: same changes as on page 515 line -1.
page 530, line 14: observe that if A is a linear operator acting in an m-dimensional Hilbert space, then $\det(I+A) = \sum_{k=0}^m \mathrm{tr}(\Lambda^k(A)) = \mathrm{tr}(\Lambda^m(I+A))$.
page 531, line 9: replace '$\mathrm{tr}(\Lambda^n(TP))$ by '$\mathrm{tr}(\Lambda^n(\mu TP))$'.
page 532, line -6: delete '$f(u)$' from the formula; in the next line Theorem 14.29 is not needed.
page 533, line 6: replace 'basis' by 'sequence'.
page 535, line 10: replace '$H^2(\mathbb{T})$ by '$H^2(\mathbb{D})$'.
page 536, line 3: here it is assumed that $n\geqslant 0$; the case $n<0$ is entirely similar.
page 540, Problem 14.11: delete 'nonzero'.
page 555, line 15: replace 'being' by 'are'.
page 557, line -2: add 'If'.
page 558, line 4: this computation is somewhat obscure and can be replaced by $(P_{\{\lambda\}}h|h) = \int_{\sigma(A)} \mathbf{1}_{\{\lambda\}}\,\mathrm{d}P_h = \mathbf{1}_{\{\lambda\}}(\lambda) = 1$.
page 559, line 3: a factor $\frac{1}{2}$ is missing in front of the matrix.
page 562, line -12: I_1 should be the closed interval $[0,2^{-n}]$.
page 563, line 7: replace 'effects' by 'effect'.
page 567, lines -7 and -5: replace '$J^\star PJ$' by '$J^\star\widetilde{P}J$'.
page 570, lines -11 and -10: replace '(F,X)' by '(F,x)'.
page 572, line -13: replace 'nz_n' by 'nz^n'; line -5: replace '$\ell^2(\mathbb{N})$' by '$H^2(\mathbb{D})$'.
page 574, line 7: replace '$M(\Omega)$' by '$M_+^1(\Omega)$'; the statement of Theorem 15.32 contains an editing error (the phrase 'each of which … by the prescription … [the formula]' should be deleted). The theorem can be phrased more transparently in terms of the affine maps $\Phi_i : \mathscr{S}(H) \to M_1^+(\Omega)$ as follows (taking into account also the above erratum): 'Let Ω be a locally compact Hausdorff space whose topology is countably generated, and suppose that $\Phi_i : \mathscr{S}(H) \to M_1^+(\Omega)$, $i\in I$, are the affine mappings associated with a family of unsharp quantum mechanical observables. Then there exists a locally compact Hausdorff space X and a family of affine maps $f_i : M_1^+(X) \to M_1^+(\Omega)$, $i\in I$, such that the following conditions hold: …' (etc.; replace all Q_i by Φ_i).
page 575, line -6: replace both 'T' by 'T_n'; line -3: replace 'span' by 'convex hull'.

page 576, line 6: a factor $\frac{1}{2}$ is missing in front of the matrix; line 13: replace 'Φ' by '$\{P_1, P_2, P_3\}$'; line 16: replace '$M_1^+ S^2) \to M_1^+\{\pm 1\})$' by '$M_1^+(S^2) \to M_1^+(\{\pm 1\})$' (same correction on page 577, line 1); a factor $\frac{1}{2}$ is missing from the last 6 displayed lines of the page (e.g., '$1 \pm \xi_j$' should read '$\frac{1}{2}(1 \pm \xi_j)$').
page 577, line 3: replace 'the probability measure on $\{\pm 1\}$ giving each point mass $\frac{1}{2}$' by 'the counting measure on $\{\pm 1\}$ giving each point mass 1'; line -10: replace 'the elements of this set' by 'the extreme points of this set'.
page 578, line 13: replace 'write a unit vector $h \in \mathbb{C}^2$ as' by 'write a pure state $|h\rangle$ as'; line 16: a factor $\frac{1}{2}$ is missing in front of the matrix; line -5: replace the second '$\frac{1}{2}(1 + \mathscr{R}x_h \cdot \mathscr{R}x_{h'})$' by '$\frac{1}{2}(1 + x_h \cdot x_{h'})$.
page 582, line -11: replace 'for $f \in L^2(\Gamma)$ and $B \in \mathscr{B}(G)$' by 'for $f \in L^2(G)$ and $B \in \mathscr{B}(\Gamma)$'.
page 583, line 5: replace '$f \in L^2(\Gamma)$' by '$\phi \in L^2(\Gamma)$'.
page 586, line 18: replace '$k \in \{0, 2^n - 1\}^d$' by '$k \in \{j2^{-n} : j = 0, 1, \dots 2^n - 1\}^d$'.
page 588, line -13: replace '$e^{-ix\cdot\xi}$' by '$e^{ix\cdot\xi}$'.
page 591, line 12: replace 'We' by 'we'.
page 595, line 7: replace '$(\widetilde{W}(a_0)h|h')$' by '$(\widetilde{W}(a_0)g|g')$'; line 8: replace $(g|g')$ by $(h|h')$'; line 9: replace '$(\widetilde{W}(a_0)h|h') =$ '$(\widetilde{W}(a_0)h|\widetilde{W}(a_0)h')$' by '$(\widetilde{W}(a_0)g|g') = (\widetilde{W}(a_0)g|\widetilde{W}(a_0)g') = (h|h')$'.
page 599, line -8: replace '$\phi_h \in L^2(\mathbb{R}^d, \gamma) = L^2(\mathbb{R}^d, \gamma)$' by '$\phi_h \in L^2(\mathbb{R}^d, \gamma) = L^2(\mathbb{R}^d, \gamma; \mathbb{K})$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$'.
page 600, lines 12 and 13: these lines can be omitted; line -10: the c_j should be real.
page 605, line -7: replace '$h^{\otimes 0} = 1$' by 'terms of the form $h_j^{\otimes 0}$ are omitted'.
page 606, line -8: replace 'c_{dj}' by 'c_{kj}'.
page 608, line 13: omit 'of $H^{\otimes n}$'.
page 609, line 13: replace '$K_{e^{-t}h}\Gamma(e^{-t}I)K_h$' by '$K_{e^{-t}h} = \Gamma(e^{-t}I)K_h$'.
page 610, lines 9 and 11: replace '$\|D_\varepsilon \xi\|$' by '$|D_\varepsilon \xi|$'.
page 613, line 10: replace the second '$a_n^\dagger(e_1)x_1$' by '$a_n^\dagger(e_d)x_d$', and replace 'x_g' by 'x_d'; lines -3 and -2: replace '$S(t)$' by '$OU(t)$'.
page 614, line 3: replace 'g' by 'g_j'; line -7: for $n = 0$ the identity $Af = \nabla f$ is trivial, so in what follows we take $n \geqslant 1$.
page 620, Problem 5.15: replace 'U is unitary on $\mathbb{C}^d$' by 'U is an isometry on $\mathbb{R}^d$'.
page 629, lines -9 to -7: the sets containing y should be called $V_{x,y}$.
page 633, line -9: replace 'on A' by 'on B'.
page 643, line -1: the closedness assumption can be omitted.
page 644, line 20: replace 'are' by 'is'; line -15: replace 'that S is totally bounded' by 'that S is complete and totally bounded'.
page 645, line -10: replace all 'N' by'd'.
page 655, line 10: replace '$(a_1, a_1] \cup \dots \cup (a_m, a_m]$' by '$(a_1, b_1] \cup \dots \cup (a_m, b_m]$'.
page 656, line -7: replace 'μ_h is a measure' by 'μ_h is a σ-finite Borel measure'.
page 659, lines 2, 3, and 5: replace '$B(x_n; \frac{1}{k})$' by '$\overline{B}(x_n; \frac{1}{k})$'; line -7: delete the second 'μ'.
page 661, line -8: replace 'functions' by 'measurable functions'.
page 663, line 14: replace '$\frac{j}{n}$' by '$\frac{j}{2^n}$'.
page 684, line 8: replace '$(-\mathrm{div} A_a \nabla)^{1/2}$' by '$A_a^{1/2}$'; line -20: delete: 'Step 1 … Mercer (1909)'.

Addenda

page 143, Theorem 4.33: Step 3 can be slightly simplified as follows. The set $C-D$ is open and convex, and from $C\cap D=\varnothing$ it follows that $0\notin C-D$. From Step 2 we obtain a functional $x^*\in X^*$ such that $0\notin\langle C-D,x^*\rangle$, which is the same as saying that $\langle C,x^*\rangle\cap\langle D,x^*\rangle=\varnothing$.

page 177, Theorem 5.2: The uniform boundedness theorem is sometimes referred to as the Banach–Steinhaus theorem.

page 177, Proposition 5.12: this can also be deduced directly from the open mapping theorem. To this end note that if $X=X_0\oplus X_1$ is a direct sum decomposition, then $|||x|||:=\|x_0\|+\|x_1\|$, with $x=x_0+x_1$ along the decomposition, defines a complete norm on X, and the mapping $x\mapsto x$ is bounded (in fact, contractive) from $(X,|||\cdot|||)$ to X by the triangle inequality. By Corollary 5.9, its inverse is bounded as well. The boundedness of the projections π_0 and π_1 immediately follows from this.

page 242, Proposition 7.26: at the end of the proof one could also observe that

$$(I+SU)^{-1}S=\sum_{n=0}^{\infty}(SU)^nS=\sum_{n=0}^{\infty}S(US)^n=S(I+SU)^{-1}$$

by the Neumann series, and then apply Atkinson's theorem.

page 479, Proposition 13.49: to check the conditions of Theorem 13.35, let us denote the Dirichlet and Neumann Laplacians by Δ. The operator $-\Delta$ is positive and selfadjoint by Theorem 12.20. This implies that $-(\Delta f|f)\in[0,\infty)$ for all $f\in\mathsf{D}(\Delta)$. Also, the spectrum of $-\Delta$ is contained in $[0,\infty)$ and therefore $I-\Delta$ is injective. Dualising and using selfadjointness, this implies that $I-\Delta$ has dense range.

page 479, Appendix B: The construction of $V\otimes W$ presented here has the advantage of connecting in an intuitive way to the various functional analytic settings where tensor products are employed. The drawback of this construction is that it only yields a nontrivial space if a supply of bilinear mappings from $V\times W$ to $\mathbb{K}$ can be guaranteed. In the same vein, the linear independence proof on the second page of the appendix relies on the availability of sufficiently many linear functionals (although this may be circumvented by passing to suitable finite-dimensional subspaces of V and W). Such supplies can be guaranteed by using Zorn's lemma, which allows one to find algebraic bases for V and W; with such bases at hand, one may use the coordinate functionals associated with these bases. In the main text, however, V and W will always be Hilbert spaces, and the required supply of bilinear and linear functionals is guaranteed through the inner product.